ZEROS OF POLYNOMIAL FUNCTIONS

Summary of Properties

1. The function given by $f(x) = a_n x^n + a_{n-1}x^{n-1} + x_{n-2}x^{n-2} + \ldots + a_2x^2 + a_1x + a_0$ is called a **polynomial function** of x with **degree n**, where n is a nonnegative integer and $a_n, a_{n-1}, a_{n-2}, \ldots, a_2, a_1, a_0$ are real numbers with a_n 0.

2. The graphs of polynomial functions are **continuous** and have no sharp corners. The sign of the **leading coefficient** a_n determines the **end behavior** of the function. The degree n determines the number of complex zeros of the function. The number of real zeros of the function will be less than or equal to the number of complex zeros.

3. The **real zeros** of a polynomial function may be found by factoring (where possible) or by finding where the graph touches the x-axis. The number of times a zero occurs is called its **multiplicity**. If a function has a zero of odd multiplicity, the graph of the function crosses the x-axis at that x-value. However, if a function has a zero of even multiplicity, the graph of the function only touches the x-axis at that x-value.

4. The graphing calculator has a built-in function for finding a **zero** (or **root**) of a function. As an alternative method, you can graph y = 0 (the x-axis) as a second function and use the **intersection** function to find the zero. While this latter method is somewhat easier to use on some calculators, it may not work for finding zeros of even multiplicity.

Finding the Zeros of Polynomial Functions

Find the real zeros and state the multiplicity of each for the following polynomial functions:

Algebraic solution — Graphical solution

1. $f(x) = 4x^2 - 3x - 7$

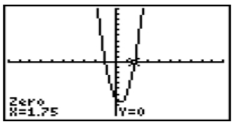
$4x^2 - 3x - 7 = 0$

$(4x - 7)(x + 1) = 0$

$4x - 7 = 0$ or $x + 1 = 0$

$x = \frac{7}{4}$ or $x = -1$

Each zero has multiplicity one.



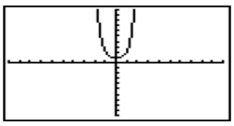
Repeat to find other zero

algebraic solution — graphical solution

2. $f(x) = x^4 + 1$

$x^4 + 1 = 0$

$x^4 = -1$ has no real solutions



This function has no real zeros.