

Chapter 8 Variation and Polynomial Equations

8-1 Direct Variation and Proportion

- 1) y varies directly with x
- 2) y varies with x
- 3) y is directly proportional to x
- 4) y is proportional to x

$$y = kx$$

8-2 Inverse and Joint Variation

- 1) y varies inversely as x
- 2) y is inversely proportional to x

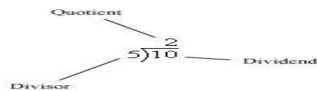
$$y = \frac{k}{x}$$

Joint Variation is two or more direct variations.

Ex. y varies jointly with x and z .

$$y = k \cdot x \cdot z$$

8-3 Dividing Polynomials



8-4 Synthetic Division

Ex 1 $(2x^3 + x^2 + 12) \div (x+2)$

$$\begin{array}{r|rrrr}
 -2 & 2 & 1 & 0 & 12 \\
 & \downarrow & -4 & 6 & -12 \\
 \hline
 & 2x^2 & -3x & +6 & 0
 \end{array}$$

$2x^2 - 3x + 6$

8-5 The Remainder and the Factor Theorems

Remainder Theorem: If $P(x)$ is a polynomial of degree n ($n \geq 0$), then for any number r ,

$P(x) = Q(x) \cdot (x-r) + P(r)$, where $Q(x)$ is a polynomial of degree $n-1$. For the polynomial $P(x)$, the function value $P(r)$ is the remainder when $P(x)$ is divided by $x-r$.

Factor Theorem: A polynomial $P(x)$ has $(x-r)$ as a factor if and only if r is a root of the equation $P(x) = 0$

8-6 Some Useful Theorem For Solving Polynomial Equations

The Fundamental Theorem of Algebra (Carl Gauss)

For every polynomial of degree $n > 1$ (with complex coefficients) there exists at least one linear factor.

Another Theorem by Carl Friedrich Gauss

Every polynomial of degree $n > 1$, (with complex coefficients) can be factored into exactly n linear factors.

Once we have these n linear factors, we can use the Zero Product Property to find the n roots or solutions of the polynomial.

Conjugate Root Theorem for Complex Roots

If a polynomial $P(x)$ of degree greater than or equal to 1 (with real coefficients) has a complex number as a root $a + bi$, then its conjugate $a - bi$ is also a root

8-7 Finding Rational Roots

Rational Root Theorem

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$$

... where all coefficients are integers.

Find a rational number c/d , where c and d are relatively prime. For c/d to be a root of $P(x)$, c must be a factor of the constant, c and d must be a factor of the leading coefficients.

Possible Rational Roots:

$$p(x) = 3x^4 - 11x^3 + 10x - 4$$

$$c : \pm 1 \pm 2 \pm 4$$

$$d : 1 \pm 3$$

$$\frac{c}{d} : \pm 1 \pm 2 \pm 4 \pm \frac{1}{3} \pm \frac{2}{3} \pm \frac{4}{3}$$