

Probability 2: *Compound Events*

Events that are independent of each other are events whose outcomes cannot affect each other. This isn't a precise definition — how can you tell whether events are independent if they involve the same people, for example? — but it will do for now. You'll get a better definition and a test for independence later in your course.

The Probability 1 worksheet discussed how to determine the probability of one random event. If you have a series of independent events, you can calculate the probability of the series from those individual events.

Example 1: Three coins are flipped: a penny, a nickel and a dime. What is the probability of getting heads on (a) the penny? (b) the nickel? (c) the dime? (d) all three?

Solution: There are two outcomes for each flip, and these outcomes are equally likely, so we can say that the answer to (a), (b) and (c) is $\frac{1}{2}$. (d) is a harder question. We go back to our definition of how to calculate the probability of a random event: For outcomes that are equally likely,

$$P(\text{event}) = \frac{\text{the total number of successes}}{\text{the total number of outcomes}}$$

We can list all the outcomes of the coin flips. There are eight of them, and only one of them is a success as the question defines it, so the answer is $\frac{1}{8}$.

This answer is also equal to $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$, the product of the answers to (a), (b) and (c). This suggests the rule:

The probability of a series of independent events is equal to the product of the probabilities of the individual events.

or: $P(A \text{ and } B) = P(A) \cdot P(B)$, where A and B are independent.

This is a different rule from the addition rule in the Probability 1 worksheet.

The addition rule deals with one event, and multiple outcomes of that event.

The multiplication rule here refers to the outcomes of a series of events.

We use "and" and "or" to describe combinations of events, but they're common words used in many contexts. It may help you to think in terms of "and then" versus "or else".

If the parts of the problem can be connected with "and then" — I flip the penny and then I flip the nickel — then every piece of the problem is another hoop to jump through, and it only gets harder to succeed. We multiply these probabilities (since multiplying fractions makes them smaller), and we check if the events are independent.

If the parts of the problem can be connected with "or else" — I flip a head on the penny or else I roll an even number on a die — then every piece of the problem is a way to succeed, and it gets easier. We add these probabilities (since adding fractions makes them bigger), and we check if the events are disjoint. (See Probability 1 for a review.)