A.P Calculus Worksheet: Areas of Surfaces of Revolution

Find the areas of the surfaces generated by revolving the curves in problems 1-4 about the axes indicated:

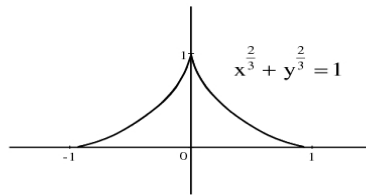1. $y = x/2$, $0 \le x \le 4$, about the x-axis. Check your result with a formula from geometry.

2. $y = \dfrac{x^3}{9}$, $0 \le x \le 2$, about the x-axis.

3. $y = \sqrt{2x - x^2}$, $0 \le x \le 2$, about the x-axis.

4. $x = \dfrac{y^3}{3}$, $0 \le y \le 1$, about the y-axis.

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5. The surface of a sphere. Use an integral to find the surface area of the sphere generated by revolving the semicircle $y = \sqrt{1 - x^2}$, $-1 \le x \le 1$, about the x-axis. Check your result with a formula from geometry.

6. The surface of an astroid. Find the area of the surface generated by revolving the portion of the astroid $x^{2/3} + y^{2/3} = 1$ shown below about the x-axis. (Hint: Revolve the first quadrant portion $y = \left(1 - x^{2/3}\right)^{3/2}$, $0 \le x \le 1$, about the x-axis, and double your result.)

$x^{\frac{2}{3}} + y^{\frac{2}{3}} = 1$

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Answers:

1. $4\sqrt{5}\pi$ 　　 2. $\dfrac{98\pi}{81}$ 　　 3. 4π 　　 4. $\dfrac{\pi\left(2\sqrt{2} - 1\right)}{9}$ 　　 5. 4π 　　 6. $\dfrac{12\pi}{5}$