

Exponential Functions

An exponential function grows (or decays) by the same *relative* amount (the same *percentage*) in each unit of time. For a quantity Q growing exponentially with a fractional growth rate r ,

$$Q = Q_0 \times (1+r)^t$$

where

Q = value of exponentially growing quantity after time t

Q_0 = initial value of the quantity

r = fractional growth rate of the quantity (this is the percentage by which the quantity is growing, converted to decimal).

t = time

Negative values of r correspond to exponential decay. Note that *the units of time used for t and r must be the same*. For example, if the fractional growth rate is 0.05 per month, then t must also be in months.

Example 1:

The population of Greenville is increasing at a rate of 5.6% per year. If the population today is 8,000, what will it be 10 years from now?

Solution:

Using our formula above with $Q_0 = 8,000$, $r = .056$ (per year), and $t = 10$ (years), we have

$$Q = Q_0 \times (1+r)^t$$

$$Q = 8,000 \times (1+.056)^{10}$$

$$Q = 13,795$$

This tells us that 10 years from now the population will be 13,795.