

Starting From \mathbb{R}^n to \mathbb{C}^n

Special Note: This assignment is due on **Wednesday**.
Deadline: 11:59 PM on **Wednesday**.
Points: 100 points.
Weight: 10% of your final grade.

1.1.1.1 **Complex Numbers**
Complex numbers are introduced as a generalization of real numbers. The real numbers are considered as a subset of the complex numbers. The complex numbers are defined as $\mathbb{C} = \{a + bi \mid a, b \in \mathbb{R}\}$, where i is the imaginary unit, $i^2 = -1$. The complex numbers form a field under addition and multiplication.

1.1.1.2 **Complex Conjugate**
The complex conjugate of a complex number $z = a + bi$ is $\bar{z} = a - bi$. The complex conjugate of a complex number z is denoted by \bar{z} . The complex conjugate of a complex number z is the complex number \bar{z} such that $z + \bar{z} = 2a$ and $z - \bar{z} = 2bi$.

1.1.1.3 **Complex Plane**
The complex plane is a 2D coordinate system where the horizontal axis is the real axis and the vertical axis is the imaginary axis. A complex number $z = a + bi$ is represented by a point (a, b) in the complex plane. The complex plane is a vector space over \mathbb{R} .

1.1.1.4 **Complex Addition and Multiplication**
Complex addition and multiplication are defined as follows:
 $(a + bi) + (c + di) = (a + c) + (b + d)i$
 $(a + bi)(c + di) = (ac - bd) + (ad + bc)i$

1.1.1.5 **Complex Division**
Complex division is defined as follows:
 $\frac{a + bi}{c + di} = \frac{(a + bi)(c - di)}{(c + di)(c - di)} = \frac{(ac + bd) + (bc - ad)i}{c^2 + d^2}$

1.1.1.6 **Complex Exponentiation**
Complex exponentiation is defined as follows:
 $e^{a + bi} = e^a (\cos b + i \sin b)$

1.1.1.7 **Complex Logarithm**
Complex logarithm is defined as follows:
 $\log z = \ln |z| + i \arg z$