

## Starting From $\mathbb{R}^n$ to $\mathbb{C}^n$

**Special Note:** This assignment is due on **Wednesday**.  
**Prerequisites:** Multivariable Calculus and Differential Equations.  
**Prerequisites:** Matrix Algebra, Complex Numbers, and Linear Algebra.  
**Prerequisites:** Complex Numbers and Linear Algebra.

### 1.1. $\mathbb{R}^n$ to $\mathbb{C}^n$

Consider the mapping  $f: \mathbb{R}^n \rightarrow \mathbb{C}^n$  defined by  $f(x) = x + iy$ , where  $x \in \mathbb{R}^n$  and  $y \in \mathbb{R}^n$ . Show that  $f$  is a linear transformation and that  $f$  is invertible. Find the matrix of  $f$  with respect to the standard bases of  $\mathbb{R}^n$  and  $\mathbb{C}^n$ .

Use the fact that  $f$  is a linear transformation to show that  $f$  is invertible. Find the matrix of  $f^{-1}$  with respect to the standard bases of  $\mathbb{R}^n$  and  $\mathbb{C}^n$ . Show that  $f$  is a linear transformation and that  $f$  is invertible. Find the matrix of  $f$  with respect to the standard bases of  $\mathbb{R}^n$  and  $\mathbb{C}^n$ .

### 1.2. $\mathbb{C}^n$ to $\mathbb{R}^n$

#### (a) $\mathbb{C}^n$ to $\mathbb{R}^n$

- 1.1.1. Show that the mapping  $f: \mathbb{C}^n \rightarrow \mathbb{R}^n$  defined by  $f(x + iy) = x - iy$  is a linear transformation. Find the matrix of  $f$  with respect to the standard bases of  $\mathbb{C}^n$  and  $\mathbb{R}^n$ .
- 1.1.2. Show that the mapping  $f: \mathbb{C}^n \rightarrow \mathbb{R}^n$  defined by  $f(x + iy) = x + iy$  is a linear transformation. Find the matrix of  $f$  with respect to the standard bases of  $\mathbb{C}^n$  and  $\mathbb{R}^n$ .
- 1.1.3. Show that the mapping  $f: \mathbb{C}^n \rightarrow \mathbb{R}^n$  defined by  $f(x + iy) = x - iy$  is a linear transformation. Find the matrix of  $f$  with respect to the standard bases of  $\mathbb{C}^n$  and  $\mathbb{R}^n$ .

#### (b) $\mathbb{R}^n$ to $\mathbb{C}^n$

- 1.2.1. Show that the mapping  $f: \mathbb{R}^n \rightarrow \mathbb{C}^n$  defined by  $f(x) = x + iy$  is a linear transformation. Find the matrix of  $f$  with respect to the standard bases of  $\mathbb{R}^n$  and  $\mathbb{C}^n$ .
- 1.2.2. Show that the mapping  $f: \mathbb{R}^n \rightarrow \mathbb{C}^n$  defined by  $f(x) = x - iy$  is a linear transformation. Find the matrix of  $f$  with respect to the standard bases of  $\mathbb{R}^n$  and  $\mathbb{C}^n$ .
- 1.2.3. Show that the mapping  $f: \mathbb{R}^n \rightarrow \mathbb{C}^n$  defined by  $f(x) = x + iy$  is a linear transformation. Find the matrix of  $f$  with respect to the standard bases of  $\mathbb{R}^n$  and  $\mathbb{C}^n$ .
- 1.2.4. Show that the mapping  $f: \mathbb{R}^n \rightarrow \mathbb{C}^n$  defined by  $f(x) = x - iy$  is a linear transformation. Find the matrix of  $f$  with respect to the standard bases of  $\mathbb{R}^n$  and  $\mathbb{C}^n$ .

#### (c) $\mathbb{C}^n$ to $\mathbb{C}^n$

- 1.3.1. Show that the mapping  $f: \mathbb{C}^n \rightarrow \mathbb{C}^n$  defined by  $f(x + iy) = x + iy$  is a linear transformation. Find the matrix of  $f$  with respect to the standard bases of  $\mathbb{C}^n$  and  $\mathbb{C}^n$ .
- 1.3.2. Show that the mapping  $f: \mathbb{C}^n \rightarrow \mathbb{C}^n$  defined by  $f(x + iy) = x - iy$  is a linear transformation. Find the matrix of  $f$  with respect to the standard bases of  $\mathbb{C}^n$  and  $\mathbb{C}^n$ .