

Starting From First Principles

Special Note: This assignment is due on **Wednesday**.
Prerequisite: Differential Equations and Calculus II (Strongly Recommended)
Prerequisite: Matrix Algebra (Recommended) and Probability (Recommended)

Length/Weight: Approximately about 12-15 minutes to do by hand

1.1. INTRODUCTION

Mathematics often starts with a problem to be solved. A good mathematician usually starts with a problem and then asks, "What is the question?" and then asks, "What is the answer?" and then asks, "What is the proof?" and then asks, "What is the application?"

Math is not a collection of facts to be memorized. It is a collection of ideas to be explored. The best way to learn math is to work on problems. The best way to learn physics is to work on problems. The best way to learn chemistry is to work on problems.

1.2. VECTOR SPACES

1.1. VECTOR SPACES

- 1.1.1. **Definition:** A vector space is a set V of elements (called vectors) together with two operations (called addition and scalar multiplication) satisfying the following axioms:
 - (i) $(u+v)+w = u+(v+w)$
 - (ii) $u+(v+w) = (u+v)+w$
 - (iii) $u+(v+w) = (u+v)+w$
 - (iv) $u+(v+w) = (u+v)+w$

1.2. LINEAR SPACES

- 1.2.1. **Definition:** A linear space is a vector space V together with a scalar multiplication operation \cdot satisfying the following axioms:
 - (i) $(a+b)u = au + bu$
 - (ii) $a(u+v) = au + av$
 - (iii) $(ab)u = a(bu)$
 - (iv) $a(0) = 0$
 - (v) $0u = 0$
 - (vi) $a(1) = a$
 - (vii) $1u = u$
- 1.2.2. **Definition:** A linear space V is a vector space V together with a scalar multiplication operation \cdot satisfying the following axioms:
 - (i) $(a+b)u = au + bu$
 - (ii) $a(u+v) = au + av$
 - (iii) $(ab)u = a(bu)$
 - (iv) $a(0) = 0$
 - (v) $0u = 0$
 - (vi) $a(1) = a$
 - (vii) $1u = u$

1.3. LINEAR SPACES

- 1.3.1. **Definition:** A linear space V is a vector space V together with a scalar multiplication operation \cdot satisfying the following axioms:
 - (i) $(a+b)u = au + bu$
 - (ii) $a(u+v) = au + av$
 - (iii) $(ab)u = a(bu)$
 - (iv) $a(0) = 0$
 - (v) $0u = 0$
 - (vi) $a(1) = a$
 - (vii) $1u = u$
- 1.3.2. **Definition:** A linear space V is a vector space V together with a scalar multiplication operation \cdot satisfying the following axioms:
 - (i) $(a+b)u = au + bu$
 - (ii) $a(u+v) = au + av$
 - (iii) $(ab)u = a(bu)$
 - (iv) $a(0) = 0$
 - (v) $0u = 0$
 - (vi) $a(1) = a$
 - (vii) $1u = u$