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Leibniz's Calculus (Real Retro Calc.)

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21.1 Introduction

To many students, differential calculus seems like a set of rules to be applied for solving problems such as optimization problems, tangent problems, etc. This really should not be surprising as differential calculus literally is a set of rules for calculating differences. These rules first appeared in Leibniz's 1684 paper *Nova methodus pro maximus et minimus, itemque tangentibus, quae nec fractus nec irrationals, quantitates moratur, et singulare pro illi calculi genus* (*A New Method for Maxima and Minima as Well as Tangents, Which is Impeded Neither by Fractional Nor by Irrational Quantities, and a Remarkable Type of Calculus for This*). A translation of this appears in [5, p. 272–80]. As the title suggests, our students' perceptions are not far off. Indeed, Leibniz's differential calculus is very recognizable to modern students and illustrates the fact that this is really a collection of rules and techniques to compute and utilize (infinitesimal) differences. The fact that Leibniz's notation is so modern in appearance, or rather our notation is that of Leibniz, allows these rules to be presented in a typical calculus class. The author has typically done this while covering the differentials section of the course, as the rules are rules for differentials, not derivatives. Doing this reinforces the rules for computing derivatives and introduces the student to the manipulation of differentials that will be necessary in integration.

A bolder approach, which the author has employed, is to replace the typical "limit of difference quotient" derivations with these heuristic arguments and adopt the point of view that $\frac{dy}{dx}$ is a ratio of infinitesimals. In this case, the derivation of the typical derivative rules can be accomplished without reliance on limits of difference quotients. The fact that these can be obtained in a more algebraic manner might be more in the comfort zone of the typical beginning calculus student.

Even in the less bold classroom, students can be made to appreciate that the modern "limit of difference quotient" definition is not how the subject was originally conceived. The fact that these difference rules allowed a systematic approach to problems in tangents and optimization led to a golden period in mathematics during the eighteenth century. Indeed, Leibniz himself applied his differential rules to study classical problems such as Snell's Law of Refraction and his quite modern approach is worth presenting. Furthermore, the exploitation of these calculus rules and the power of differentials can be seen in Johann Bernoulli's 1696 solution of the brachistochrone problem. This masterpiece of mathematical work certainly deserves a place in any calculus class and can easily be included at the end of an AP calculus course.

This article includes these applications along with exercises which will enhance students' heuristic understanding of Leibniz's calculus rules and fluency with differentials. More applications of differentials can be found in [3, p.