

MA 12 LG 15 (Combinatorics)

1.1. The Fundamental Counting Principle

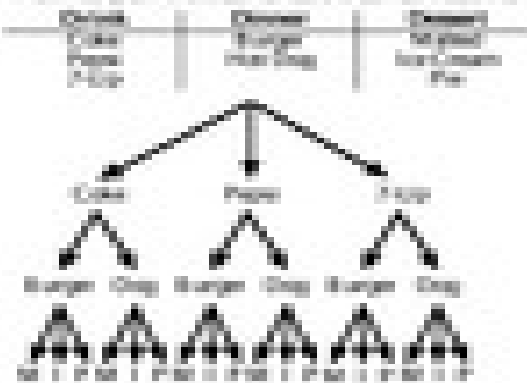
If P and Q are events such that

where L, M, N are the number of items in each category

Example for 1.1

Draw a tree diagram to list all the possibilities for the following:

How many different combinations can be made from



Total possibilities = $3 \times 2 \times 2 = 12$ possibilities

1.2. Permutations

General: $n! = n(n-1)(n-2)\dots(2)(1) = n \times (n-1) \dots 1$

Permutations (order is important) are different from sets

Permutations = $\frac{n!}{(n-r)!}$ n = total # objects
 r = # to select r = # of objects after

$${}^n P_r = \frac{n!}{(n-r)!}$$

Example for 1.2

3 spaces in factorial notation:

the 3 is the # of the 3's, the 2 is the # of 2's

Example for 1.1

3 spaces in exponent of factorial $n!$ in notation/terms:

$3! = 3 \times 2 \times 1 = 2 \times 2 \times 1$

$$\begin{aligned}
 {}^3 P_2 &= \frac{3!}{(3-2)!} = \frac{3!}{1!} \\
 &= \frac{3 \times 2 \times 1}{1 \times 1} \\
 &= \frac{3 \times 2}{1} \\
 &= 6 \text{ possibilities}
 \end{aligned}$$

Example for 1.2

3 spaces: 12 is 12, 11 is 11, 10 is 10, etc. for the form ${}^n P_r$

$${}^3 P_2 = \frac{3!}{(3-2)!}$$

12 is 12, 11 is 11, etc. \rightarrow $\frac{12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}$

$$\begin{aligned}
 n = 12, \quad r = 2 \\
 \frac{12!}{10!} &= \frac{12 \times 11 \times 10!}{10!} \\
 &= 12 \times 11 \\
 &= 132
 \end{aligned}$$

1.3. Combinations

General: $n! = n(n-1)(n-2)\dots(2)(1) = n \times (n-1) \dots 1$

$${}^n C_r = \frac{n!}{r!(n-r)!}$$

Combinations (order not important) are different from sets

$${}^n C_r = \frac{n!}{r!(n-r)!}$$

Example for 1.3

3 spaces in exponent in notation/terms:

$$\begin{aligned}
 {}^3 C_2 &= \frac{3!}{2!(3-2)!} = \frac{3!}{2!1!} \\
 &= \frac{3 \times 2 \times 1}{2 \times 1 \times 1} \\
 &= \frac{3 \times 2 \times 1}{2 \times 1} \\
 &= \frac{3 \times 2}{2} \\
 &= 3 \text{ possibilities}
 \end{aligned}$$

$$\begin{aligned}
 {}^3 C_1 &= \frac{3!}{1!(3-1)!} = \frac{3!}{1!2!} \\
 &= \frac{3 \times 2 \times 1}{1 \times 2 \times 1} \\
 &= \frac{3 \times 2 \times 1}{2 \times 1} \\
 &= \frac{3 \times 2}{2} \\
 &= 3 \text{ possibilities}
 \end{aligned}$$