Trigonometric Identities

The nature of trigonometric ratios means that there are many expressions that are equivalent to each other even if they look nothing alike. These expressions are called **trigonometric identities**. Proving a trigonometric identity simply means demonstrating that the two expressions really are equivalent.

There's no pattern or algorithm for doing proofs like these. There are a couple of strategies, though. It is better to start with simplifying the more complicated side so that it looks more like the simpler side. Look for parts of one side or the other that appear in your standard trig identities (so that if you see "tan² θ", you might look for a way to use the Pythagorean identity that mentions it). Factoring an expression is sometimes helpful. And if all else fails, you can express everything in terms of sines and cosines.

Example 1:   Prove: $\csc x - \cos x \cot x = \sin x$

Solution: LHS: $\csc x - \cos x \cot x = \dfrac{1}{\sin x} - \cos x \dfrac{\cos x}{\sin x}$

$$= \frac{1 - \cos^2 x}{\sin x}$$

$$= \frac{\sin^2 x}{\sin x}$$

$$= \sin x = \text{RHS} \quad \checkmark$$

Example 2:   Prove: $\dfrac{1 + \cos 2\theta}{\sin 2\theta} = \cot \theta$

Solution: Since we see "2θ" on the left side, and no "2θ" on the right, our first step should be to use the double angle identities. We'll try the form of the cos 2θ identity that gets rid of the 1 in the numerator (this might be a bad choice, but the only way to know is to try it):

$$\text{LHS: } \frac{1 + \cos 2\theta}{\sin 2\theta} = \frac{1 + (2\cos^2 \theta - 1)}{2 \sin \theta \cos \theta}$$

$$= \frac{2 \cos^2 \theta}{2 \sin \theta \cos \theta}$$

$$= \frac{\cos \theta}{\sin \theta}$$

$$= \cot \theta = \text{RHS} \quad \checkmark$$