

$$x^2 \frac{dy}{dx} + 3xy = 2 + x$$

$$\frac{dy}{dx} + \frac{3xy}{x^2} = \frac{2}{x^2} + \frac{x}{x^2}$$

$$\frac{dy}{dx} + \left(\frac{3}{x}\right)y = \frac{2}{x^2} + \frac{1}{x} \quad (i)$$

remember

$$\frac{dy}{dx} + Py = Q$$

$$\therefore P = \frac{3}{x}, \quad Q = \frac{2}{x^2} + \frac{1}{x}$$

multiplying (i) by x^3 to make the LHS like a Product Rule result

$$x^3 \frac{dy}{dx} + 3x^2y = 2x + x^2$$

but $\frac{d(x^3y)}{dx} = x^3 \frac{dy}{dx} + 3x^2y$

$$\therefore \frac{d(x^3y)}{dx} = 2x + x^2$$

integrating w.r.t. x

$$\int \frac{d(x^3y)}{dx} = \int (2x + x^2) dx$$

$$x^3y = \frac{2x^2}{2} + \frac{x^3}{3} + C$$

$$x^3y = x^2 + \frac{x^3}{3} + C$$
