

$$\begin{aligned}
\int_0^{\infty} \frac{dx}{(x+1)\sqrt{x}} &= \lim_{s \rightarrow 0} \int_s^1 \frac{dx}{(x+1)\sqrt{x}} + \lim_{t \rightarrow \infty} \int_1^t \frac{dx}{(x+1)\sqrt{x}} \\
&= \lim_{s \rightarrow 0} \left( \frac{\pi}{2} - 2 \arctan \sqrt{s} \right) + \lim_{t \rightarrow \infty} \left( 2 \arctan \sqrt{t} - \frac{\pi}{2} \right) \\
&= \frac{\pi}{2} + \left( \pi - \frac{\pi}{2} \right) \\
&= \frac{\pi}{2} + \frac{\pi}{2} \\
&= \pi.
\end{aligned}$$