

13. (10pts) Given that $x^2 + 4$ is a factor, factor the polynomial $x^4 - 3x^3 + 6x^2 - 12x + 8$ completely, using complex coefficients if necessary.

$$\begin{array}{r}
 x^2 - 3x + 2 \\
 x^2 + 4 \overline{) x^4 - 3x^3 + 6x^2 - 12x + 8} \\
 \underline{x^4 + 4x^2} \\
 -3x^3 + 2x^2 - 12x + 8 \\
 \underline{-3x^3 - 12x} \\
 2x^2 + 8 \\
 \underline{2x^2 } \\
 0
 \end{array}$$

$$(x^2 + 4)(x^2 - 3x + 2) \\
 (x + 2i)(x - 2i)(x - 1)(x - 2)$$

14. (10pts) For the quadratic function $f(x) = 2x^2 + 8x + 7$

(a) determine the vertex of the parabola defined by the function $f(x)$;

$$\begin{array}{l}
 \text{Vertex} \\
 \text{at}
 \end{array}
 \quad x = -\frac{b}{2a} = -\frac{8}{2 \cdot 2} = -2 \qquad (-2, -1)$$

$$f(-2) = 2(-2)^2 + 8(-2) + 7 = -1$$

(b) determine all x -intercepts (with exact values) of the graph of $f(x)$.

$$2x^2 + 8x + 7 = 0$$

$$x = \frac{-8 \pm \sqrt{8^2 - 4 \cdot 2 \cdot 7}}{2 \cdot 2} = \frac{-8 \pm \sqrt{8}}{4} = -\frac{4 \pm \sqrt{2}}{2}$$

$$\left(-\frac{4 + \sqrt{2}}{2}, 0 \right) \\
 \left(-\frac{4 - \sqrt{2}}{2}, 0 \right)$$

15. (10pts) Completely expand the logarithm as sum, difference, and/or constant multiples of simple logarithms ($x > 1$).

$$\ln \sqrt{x^3(x^2 - 1)} = \ln \left(x^3(x^2 - 1) \right)^{1/2}$$

$$= \frac{1}{2} \ln (x^3(x^2 - 1)) = \frac{1}{2} \ln x^3 + \frac{1}{2} \ln (x^2 - 1)$$

$$= \frac{3}{2} \ln x + \frac{1}{2} \ln (x + 1) + \frac{1}{2} \ln (x - 1)$$

16. (10pts) Solve the equation $e^{2x} - 6e^x + 5 = 0$ for x .

$$(e^x)^2 - 6(e^x) + 5 = 0$$

$$(e^x - 5)(e^x - 1) = 0$$

$$e^x = 5 \quad \text{or} \quad e^x = 1$$

$$x = \ln 5 \quad \text{or} \quad x = \ln 1 = 0$$