

Questions to Ask Yourself to Solve Related Rate Problems

Question 1: Will drawing a diagram help me understand the situation?

Question 2: What are the data? What information do you have from the statement of the problem?

Related rate problems **always** involve **two or more** quantities that **change with time**.

The problem may also involve some constants and some values of the dependent variables at a specific time. Be careful not to confuse constants and values of variable quantities at a specific time.

What are the constants in the problem?

Related rate problems **always** include information about the **rate of change** of one or more of the variables.

What quantities are changing over time? What information about their rates of change is given?

Figure out what information about the rates of change of these quantities is given. In some problems you are given a rate of change that is true for an interval of time. In others you are given the rate of change at a particular instant of time. Decide which situation you have.

Question 3: What is the unknown? What information are you asked to find?

A related problem **always** asks you to find the rate of change of one or more quantities from information about the rates of change of other quantities.

What variable quantity (or quantities) is the problem asking about?

Question 4: What notation would help? Introduce suitable notation.

Give the quantities that change with time function names. Include the independent variable in the function name. E.g., if a volume changes with time, denote this volume by $V(t)$ and if a distance changes with time, denote it $d(t)$.

If you are given a rate of change at a certain instant of time, give that instant a name – t_0 or τ or some other letter you like better. Use this letter with function notation to write given information mathematically. E.g. if you are to find the rate of change of the volume at the time when the diameter is 12, denote this instant of time as t_0 and the diameter at this time as $d(t_0) = 12$.

CAUTION: There may be numerical information in the problem that does not directly involve these functions and their derivatives. Save it for later. You will probably need it to answer Question 5.

Question 5: How could you write the data (known information) using the notation you introduced?

How can you write the unknown using the notation you introduced?

Can you state the problem using this notation?

(If you can't do all these things, reexamine the notation.)

A related problem **always** asks you to find the rate of change of one of the functions from information about the rates of change of other functions. Figure out which functions (variables) the problem is asking about. State the question in terms of a derivative -- e.g. "we must find $V'(t_0)$ ".

Question 6: What are some connections between the knowns and the unknown?

(A diagram may help you recognize connections. Use the notation you introduced to write down these connections.)

Which of these connections might you use to find the unknown?

Use this connection to find the unknown.

Find an equation that relates the function or functions (variables) whose rate of change is given to the function whose rate of change you are asked to find. This equation will often come from an area, volume, distance, or trigonometric formula or from the similarity of triangles. The equation must be true for all (relevant) values of time (t) (not just the instant t_0 or τ for which you have specific numerical information). This may be the only place you use some of the numerical information given in the problem.

Differentiate with respect to t . You will almost always need the Chain Rule!

Substitute numerical information. Evaluate to find the rate of change you are looking for.

Question 7: Look back. Does the answer seem reasonable?

What strategies did you use that you might be able to use in another situation?
