L 3.1   The symbolic solution of $y' = -2xy$, $y(0) = 2$ is $y = 2e^{-x^2}$. Derive it and display a full answer check

Solution

$y' + 2xy = 0$		std form $y' + py = r$
$\dfrac{(wy)'}{w} = 0$		$w = $ integrating factor $e^{\int p \, dx}$ $w = e^{x^2}$
$(wy)' = 0$		
$wy = c$		Quadrature
$y = c\, \bar{e}^{x^2}$		$1/w = e^{-x^2}$
$2 = c\, e^0$		use $y(0) = 2$
$\boxed{y = 2\bar{e}^{x^2}}$		candidate solution

Answer check

$$LHS = y'$$
$$= (2e^{-x^2})'$$
$$= 2(-2x)e^{-x^2} \qquad \text{use } (e^u)' = u' e^u$$
$$= (-2x)\, 2\bar{e}^{x^2}$$
$$= (-2x)\, y$$
$$= RHS \qquad \text{DE verified}$$

$$LHS = y(0)$$
$$= 2\bar{e}^{x^2}\Big|_{x=0}$$
$$= 2e^0$$
$$= 2$$
$$= RHS \qquad \text{Ic verified}$$