

MA 12 LG 15 (Combinatorics)

1.1. The Fundamental Counting Principle

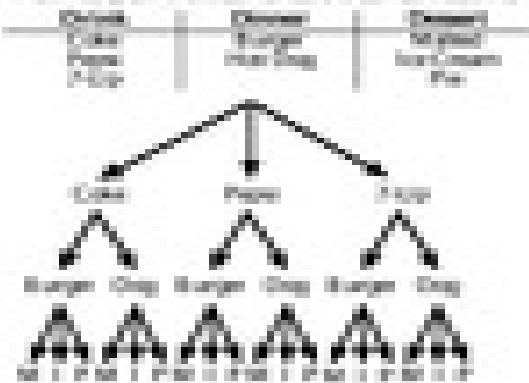
If P and Q are disjoint sets, then

where $n(P)$, $n(Q)$ are the number of items in each category.

Example for 1.1

Draw a tree diagram to list all the possibilities for the following:

How many different combinations can be made from



Total possibilities = $3 \times 2 \times 3 = 18$ possibilities

1.2. Permutations

General: $n! = n(n-1)(n-2)\dots(2)(1) = n \times (n-1) \dots 1$

Permutations (order is important) are different from sets

Permutations = $\frac{n!}{(n-r)!}$ n = total # objects
 r = # to select r = # to select = Total objects after

$${}^n P_r = \frac{n!}{(n-r)!}$$

Example for 1.2

3 objects in factorial notation:

the $3!$ is $3 \times 2 \times 1 = 6 = 3 \times 2 \times 1 = 6$

Example for 1.1

3 objects in a product of factorial $n!$ or factorial notation:

$$3! = 3 \times 2 \times 1 = 6$$

$${}^3 P_3 = \frac{3!}{(3-3)!} = \frac{3!}{0!} = \frac{3!}{1} = 3! = 3 \times 2 \times 1 = 6$$

Example for 1.2

3 objects, 2 in $3!$ or $2!$ or $1!$ or the form ${}^n P_r$

$${}^3 P_2 = \frac{3!}{(3-2)!} = \frac{3!}{1!} = \frac{3 \times 2 \times 1}{1} = 3 \times 2 = 6$$

$${}^3 P_1 = \frac{3!}{(3-1)!} = \frac{3!}{2!} = \frac{3 \times 2 \times 1}{2 \times 1} = 3$$

$${}^3 P_0 = \frac{3!}{(3-0)!} = \frac{3!}{3!} = \frac{3 \times 2 \times 1}{3 \times 2 \times 1} = 1$$

1.3. Combinations

General: $n! = n(n-1)(n-2)\dots(2)(1) = n \times (n-1) \dots 1$

$${}^n C_r = \frac{n!}{r!(n-r)!}$$

Combinations (order not important) are different from sets

$${}^n C_r = \frac{n!}{r!(n-r)!} = \frac{n!}{r!(n-r)!}$$

Example for 1.3

3 objects in a product of factorial notation:

$${}^3 C_3 = \frac{3!}{3!(3-3)!} = \frac{3!}{3!0!} = \frac{3!}{3!1} = \frac{3 \times 2 \times 1}{3 \times 2 \times 1} = 1$$

the $3!$ is $3 \times 2 \times 1 = 6 = 3 \times 2 \times 1 = 6$

$${}^3 C_2 = \frac{3!}{2!(3-2)!} = \frac{3!}{2!1!} = \frac{3 \times 2 \times 1}{2 \times 1 \times 1} = 3$$

the $2!$ is $2 \times 1 = 2 = 2 \times 1 = 2$