MA 12 LG 15 (Combinatorics)

1.1 The Fundamental Counting Principle

of combinations = a x b x c x ...

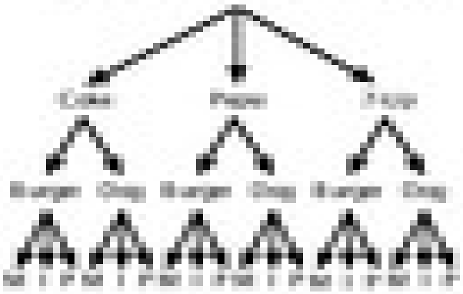
where a, b, c, are the number of choices in each category

Example for 1-4

Draw a tree diagram to list all the possibilities for the following:

How many different combinations can be made from:

Drink	Dinner	Dessert
Coke	Burger	Yogurt
Pepsi	Hot Dog	Ice Cream
7-Up		Pie



# of combos = 3 x 2 x 3 = 18 combos

1.2 Permutations

Factorial $n! = n(n-1)(n-2)(n-3)\ldots(1)$

Permutations (order is important, abc is different from acb)

Permutations = $\frac{n!}{a!b!c!\ldots}$ n = # of objects

a, b, c, ... = # of objects alike

$_nP_r = \frac{n!}{(n-r)!}$

Example for 1-8

Express in factorial notation:

8 x 7 x 6 x 5 x 4 x 3 x 2 x 1 = 8!

Example for 1-11

Express as a product of natural #'s in reduced form:

5! = 5 x 4 x 3 x 2 x 1

$_{10}P_7 = \frac{10!}{(10-7)!} = \frac{10!}{3!}$

$= \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{3 \times 2 \times 1}$

$= 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4$

Example for 16-20

Express 12 x 11 x 10 x 9 in the form $_nP_r$

$_nP_r = \frac{n!}{(n-r)!}$

$12 \times 11 \times 10 \times 9 = \frac{12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}$

$= \frac{12!}{8!} = \frac{12!}{(12-4)!}$

n = 12 r = 4 $= {}_{12}P_4$

1.3 Combinations

Factorial $n! = n(n-1)(n-2)(n-3)\ldots(1)$

$_nP_r = \frac{n!}{(n-r)!}$

Combinations (order not important, abc is same as acb)

$_nC_r = \frac{_nP_r}{r!} = \frac{n!}{(n-r)!r!}$

Example for 1-6

Express as a product in reduced form:

$_{10}P_5 = \frac{10!}{(10-5)!} = \frac{10!}{5!}$

$= \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{5 \times 4 \times 3 \times 2 \times 1}$

$= 10 \times 9 \times 8 \times 7 \times 6$

= 30240

$_{10}C_5 = \frac{10!}{(10-5)!5!} = \frac{10!}{5!5!}$

$= \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{5 \times 4 \times 3 \times 2 \times 1 \times 5 \times 4 \times 3 \times 2 \times 1}$

$= \frac{9 \times 8 \times 7 \times 6}{4 \times 3}$

$= 9 \times 7 \times 2 \times 2$

= 252