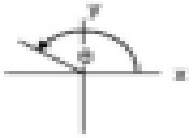
Directions of Travel

Many problems in trigonometry and physics involve objects that are moving in straight lines, but not at neat 90° angles to each other. There are many ways to express these directions. Some of them are standards in math and science, and others are used in real-life situations in various occupations. This worksheet will help you to distinguish between them and use them in problems.

DIRECTED ANGLES

Directed angles are used in trigonometry. It's a standard that derives from the unit circle and makes use of the coordinate system of the plane. These angles are expressed in radians.

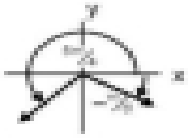


To find a directed angle, start from the **positive x-axis** and move **counterclockwise**. The positive x-axis marks 0 rad, and the negative x-axis marks π rad. 2π takes us back to the positive x-axis.

Example 1: Draw an angle of a) $\frac{7}{4}$ rad; b) $-\frac{\pi}{6}$ rad; c) $\frac{20\pi}{3}$ rad.

Solution: When you start learning how to work with radians, it may be useful to convert these angles to degrees, but it's better to practice thinking in radians. We can start by figuring out how our angle compares to π and 2π so that we have some idea what the angle looks like.

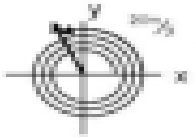
a) $\frac{7}{4}$ is higher than π but less than 2π (because $\frac{4}{4}$ is higher than 1 but less than 2), so the angle lies below the x-axis. Since $\frac{7}{4}$ is equivalent to $1\frac{3}{4}$, the angle is π plus $\frac{3}{4}$ of the way to 2π.



b) $-\frac{\pi}{6}$ is less than 0, so instead of moving counterclockwise, we move clockwise the same amount we'd be moving for $+\frac{\pi}{6}$. This is one of the angles we should know. $-\frac{\pi}{6}$ is equivalent to 30° below the positive x-axis.

c) $\frac{20\pi}{3}$ is larger than 2π (= $\frac{6\pi}{3}$), so this angle is equivalent to some other angle between 0 and 2π, which is the range we usually use with angles in radians. We can find out what that angle is by subtracting 2π from the angle until our answer is within this range (or adding 2π to the angle if the angle we're given is negative). The answer we get is called the **reference angle**.

$$\frac{20\pi}{3} - 3 \times \frac{6\pi}{3} = \frac{20\pi}{3} - \frac{18\pi}{3} = \frac{2\pi}{3}$$



The reference angle is between 0 and π, so it's above the x-axis, $\frac{2}{3}$ of the way to π. (This is another angle whose measure in degrees we should know: it's 120°.)

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