

Properties of Real Numbers: The Commutative, Associative, and Distributive Laws (1.7)**Commutative Law of Addition**

$$a + b = b + a$$

When *adding* numbers, order does not matter.

For example, $2 + 3$ is the same as $3 + 2$.

Commutative Law of Multiplication

$$ab = ba$$

When *multiplying* numbers, order does not matter.

For example, $2 \cdot 3$ is the same as $3 \cdot 2$.

Associative Law of Addition

$$a + (b + c) = (a + b) + c$$

When *adding* numbers, they can be “grouped” in any way.

For example, $(2 + 3) + 4$ gives the same result as $2 + (3 + 4)$.

Associative Law of Multiplication

$$a(bc) = (ab)c$$

When *multiplying* numbers, they can be “grouped” in any way.

For example, $2 \cdot (3 \cdot 4)$ gives the same result as $(2 \cdot 3) \cdot 4$.

Note: Subtraction and division are NOT commutative or associative operations!

Distributive Law

$$a(b + c) = ab + ac$$

The Distributive Law provides a useful way of removing parentheses

$2(x+3)$ can be written as $2 \cdot x + 2 \cdot 3 = 2x + 6$. It also allows us to

“factor” an expression: $3x + 21$ can be written as $3 \cdot x + 3 \cdot 7$

which can then be written as $3(x + 7)$

Examples

Use the Commutative Law of Addition to write an equivalent expression for $5 + 2x$.

The Commutative Law says we can change the ORDER of the addition so $2x + 5$ would be an equivalent expression.

Which law is being used here? $x(2 + z) = (2 + z)x$

Be careful! What changed from one side of the equals to the other? All that changed was the ORDER of the multiplication of the x and the $(2 + z)$. This is an example of the Commutative Law of Multiplication.

Use the Associative Law of Addition to write an equivalent expression for $(2x + 3y) + 5$.

The Associative Law says we may change the GROUPING of the addition. Let’s group the $3y$ with the 5 instead of with the $2x$. We will keep the ORDER the same (if we changed the order, we would be using the Commutative Law). $2x + (3y + 5)$ is an equivalent expression.

Use the Distributive Law to remove parentheses from $4(2x + 3)$.

$$4(2x + 3)$$

$$4 \cdot 2x + 4 \cdot 3$$

$$8x + 12$$

BOTH terms of the sum must be multiplied by 4.

Simplify when possible.

Use the Distributive Law to factor the expression $5x + 10y$

$$5x + 10y$$

$$5 \cdot x + 5 \cdot 2y$$

$$5(x + 2y)$$

We must rewrite each term with a common multiplier (5).

“Factor out” the 5 and rewrite according to the Distributive Law.