# Inequalities and the Number Line

The number line is a useful tool for both understanding and expressing inequalities. Recall that the number line is a horizontal line with tick marks showing number values. Usually we only mark integers on the number line, but we can express any real number as a point on the line (whether it is on a tick mark or falling between the tick marks). The number line below has a closed circle at the 3 tick mark. Another way we might express this is using an algebraic equation, like $x = 3$. There is only one value for x.

When x is not a single value but a range of values, we can also use the number line to show multiple solutions for x. Let's say x is any number between 0 and 3, including 0 and 3. First, mark the minimum and maximum boundaries on the number line with closed circles for now.
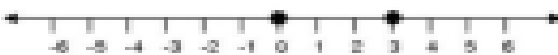
To indicate that x can be any number between 0 and 3, we imagine that we put a point at every single value that is between 0 and 3... eventually it would get so crowded that it makes more sense to just draw a line between the two boundary values. Now that we know the range of values the inequality covers, we use the standard notation of square brackets to indicate the boundary values zero and three are included in the solution. The brackets face inwards, enclosing the range of x values.

To write this type of inequality algebraically, we use the form $a \leq x \leq b$, where a and b are any numbers and a is less than b. In this case, we would write $0 \leq x \leq 3$.

We could also have a case where x can be any number between 0 and 3, but not including 0 and 3. What would this look like? On the number line, a rounded bracket means that value is the "boundary", but it's not possible for x to be exactly equal to that value. We would redraw the graph from above with rounded brackets at the two ends of the range. The inequality expression now looks like: $0 < x < 3$.

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