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CAPE Mathematics Unit 1
Worksheet 1: The Real Number System

1. Given that $p = \sqrt{2}$ and $q = \sqrt{3}$, express $\frac{\sqrt{50} - \sqrt{12}}{\sqrt{8} + \sqrt{75}}$ in terms of p and q as simply as possible.

2. Simplify these expressions by rationalising their denominators.

(a) $\sqrt{\frac{1}{8}}$ (b) $\frac{5}{\sqrt{3}-1}$ (c) $\frac{\sqrt{3}+1}{\sqrt{3}-1}$ (d) $\frac{3\sqrt{5}-4}{2\sqrt{5}+1}$

3. Simplify the following, leaving your answers in surd form:

(a) $(2\sqrt{5}+1)(3\sqrt{5}-2)$ (b) $(\sqrt{2}-1)^2(3\sqrt{2}+5)$

4. Find $\sum_{r=n+1}^{2n} r^2$ in terms of n.

5. Without using calculators or tables, show that $\sqrt{11} + \sqrt{7} = \frac{4}{\sqrt{11} - \sqrt{7}}$.

6. Prove, by Mathematical Induction, that $n^2 > 2n$ for all integers $n \geq 3$.

7. Without using calculators or tables, evaluate $(\sqrt{2}+1)^3 - (\sqrt{2}-1)^3$.

8. Prove, by Mathematical Induction, that:

(i) $10^n - 1$ is divisible by 9 for all positive integers n.

(ii) $8^n + 6$ is divisible by 14 for all positive integers n.

9. Let $S_n = \sum_{r=1}^{n} r$ for $n \in \mathbf{N}$. Find the value of n for which $3S_{2n} = 11S_n$.

Note: $\sum_{r=1}^{n} r = \frac{1}{2}n(n+1)$

10. Calculate $\sum_{r=1}^{40} (3r-1)^2$.

11. Prove by Mathematical Induction and also by using the sigma method:

(i) $\sum_{r=1}^{n} (4r-1) = n(2n+1)$

(ii) $\sum_{r=1}^{n} r(r+1)(r+3) = \frac{1}{12}n(n+1)(n+2)(3n+13)$

12. Prove by Mathematical Induction: $\sum_{r=1}^{n} \frac{1}{(3r-2)(3r+1)} = \frac{n}{3n+1}$