Stochastic Models

Simulating Markov Chains in discrete time

1. The transition matrix

A general discrete-time Markov chain has transition probabilities which are likely to vary depending on the current state. It is not too hard to write a macro in Visual Basic to carry out the transitions, but it is more challenging, and in some ways more transparent, to try to arrange a simulation mechanism using only the functions which are built into Excel. We shall use look-up tables to accomplish our goals.

Load up the workbook from the previous session and insert a new worksheet.[1] Give it some suitable name by double-clicking the tab labelled "Sheet4" (or whatever) and overtyping the new name. Enter a title for the sheet somewhere on row 1.

We are going to simulate a no-claims discount scheme with 4 states: No discount, small discount, medium discount and large discount. The transition mechanism is: if there are no claims during a year, the policyholder moves to the next higher level of discount (unless already receiving Large discount); if there is one or more claim during a year the policyholder moves to the next lower discount level (unless receiving no discount).

We need a parameter: type "Claim frequency" into cell A3, with some value like 0.4 in B3: this represents the average number of claims per year. The Poisson distribution will be used, so the probability of no claims, q, is given by the formula `=EXP(-B3)`: enter this formula in B4, with the label q in A4, and name the cell q (using Insert | Name | Define).

Now enter the transition matrix into a blank area on the worksheet, starting from column E. The diagram below contains labels in column D: they are not necessary, but serve to remind us which state is which. The procedure outlined here only works if the states are numbered with integers starting from 0. Don't forget that every entry must be non-negative and that the row sums must be 1: it's probably best to make the last column contain 1–(sum of previous columns).

	D	E	F	G	H	I	J
3	No discount	0	=1-q	=q	0	=1-SUM(F3:H3)	
4	Small disc	1	=1-q	0	=q	=1-SUM(F4:H4)	
5	Med disc	2	0	=1-q	0	=1-SUM(F5:H5)	
6	Large disc	3	0	0	=1-q	=1-SUM(F6:H6)	

The lookup function needs a cumulative transition matrix: the table shown below will calculate these in the form required. Use Insert | Name | Define to name the cumulative matrix (in this example, F8:J11) CumuMatrix, and the vector of states (F12:J12) StateVector.

	D	E	F	G	H	I	J
7							
8	No discount	0	0	=F3	=G8+G3	=H8+H3	=I8+I3
9	Small disc	1	0	=F4	=G9+G4	=H9+H4	=I9+I4
10	Med disc	2	0	=F5	=G10+G5	=H10+H5	=I10+I5
11	Large disc	3	0	=F6	=G11+G6	=H11+H6	=I11+I6
12			=E8	=E9	=E10	=E11	0

[1] The reason for reloading the workbook from session 1 is that we want to re-use the random numbers which we generated last time. If you no longer have session 1's workbook, start a new one and use the formula `=RAND()` to generate a column of 200 random numbers on one of the sheets: name this column Random. See Appendix on page 4.