VANCOUVER COMMUNITY COLLEGE

Easy Integrals (That Don't Look Easy)

Sometimes it is possible to simplify a function in an integral before you integrate it, making a difficult problem into a simple one.

Example 1: Evaluate the integrals: a) $\int (x^4-5)^2\,dx$ b) $\int (x^3+x)(2x^2-4)\,dx$

c) $\int \frac{x^3+4x-3}{x}\,dx$ d) $\int \cos x\tan^2 x+\cos x\,dx$ e) $\int \frac{1}{e^{2x}}\,dx$

Solution: a) If there was an x^3 in the integral, we could integrate by parts… but it is much easier to turn this into an ordinary polynomial:

$$\begin{aligned}\int (x^4-5)^2\,dx &= \int (x^4-5)(x^4-5)\,dx\\ &= \int x^8-10x^4+25\,dx\\ &= \tfrac{1}{9}x^9-2x^5+25x+c\end{aligned}$$

b) Same idea again. Just expand:

$$\begin{aligned}\int (x^3+x)(2x^2-4)\,dx &= \int 2x^5-2x^3-4x\,dx\\ &= \tfrac{1}{3}x^6-\tfrac{1}{2}x^4-2x^2+c\end{aligned}$$

c) And the same idea again. Divide the denominator into the numerator and the result will be a polynomial:

$$\begin{aligned}\int \frac{x^3+4x-3}{x}\,dx &= \int \frac{x^3}{x}+\frac{4x}{x}-\frac{3}{x}\,dx\\ &= \int x^2+4-3x^{-1}\,dx\\ &= \tfrac{1}{3}x^3+4x-3\ln|x|+c\end{aligned}$$

d) In this case, we can use trigonometric identities to make the integral easier:

$$\begin{aligned}\int \cos x\tan^2 x+\cos x\,dx &= \int (\cos x)(\tan^2 x+1)\,dx\\ &= \int (\cos x)(\sec^2 x)\,dx\\ &= \int \frac{\cos x}{\cos^2 x}\,dx\\ &= \int \sec x\,dx\\ &= \ln|\sec x+\tan x|+c\end{aligned}$$

e) We can also express fractions with negative exponents:

$$\begin{aligned}\int \frac{1}{e^{2x}}\,dx &= \int e^{-2x}\,dx\\ &= -\tfrac{1}{2}e^{-2x}+c\end{aligned}$$

Authored by Darren Rigby