Worksheet 9

Double pendulum

9.1 The basic task

Write a program to integrate the equations of motion for the double pendulum. Explore the solution space.

9.2 Introduction

The equations of motion of the double pendulum have been derived in class, and are an example of such equations that are easily derived from a Lagrangian, but are difficult to obtain using simpler approaches. For small amplitudes, we obtain two frequencies for the pendulum. For larger oscillations, the motion is chaotic and quasiperiodic.

9.3 Theory

The course notes give the following second order differential equations:

$$(m_1 + m_2)l_1^2\ddot{\phi}_1 + m_2 l_1 l_2 \ddot{\phi}_2 \cos(\phi_1 - \phi_2) + m_2 l_1 l_2 \dot{\phi}_2^2 \sin(\phi_1 - \phi_2) + (m_1 + m_2) g l_1 \sin\phi_1 = 0 \tag{9.1}$$

$$m_2 l_2^2 \ddot{\phi}_2 + m_2 l_1 l_2 \ddot{\phi}_1 \cos(\phi_1 - \phi_2) - m_2 l_1 l_2 \dot{\phi}_1^2 \sin(\phi_1 - \phi_2) + m_2 g l_2 \sin\phi_2 = 0 \tag{9.2}$$

These equations are coupled and not in a suitable form for numerical solution. Put $\mu = 1.0 + m_1/m_2$ and $\lambda = l_2/l_1$. Then the equations become

$$\mu\ddot{\phi}_1 + \lambda\ddot{\phi}_2 \cos(\phi_1 - \phi_2) + \lambda\dot{\phi}_2^2 \sin(\phi_1 - \phi_2) + (\mu g/l_1)\sin\phi_1 = 0 \tag{9.3}$$

$$\lambda\ddot{\phi}_2 + \ddot{\phi}_1 \cos(\phi_1 - \phi_2) - \dot{\phi}_1^2 \sin(\phi_1 - \phi_2) + (g/l_1)\sin\phi_2 = 0 \tag{9.4}$$

These equations can then be solved for the angular accelerations:

$$\ddot{\phi}_1 = \frac{g(\sin\phi_2 \cos(\phi_1 - \phi_2) - \mu \sin\phi_1) - (l_2\dot{\phi}_2^2 + l_1\dot{\phi}_1^2 \cos(\phi_1 - \phi_2))\ \sin(\phi_1 - \phi_2)}{l_1(\mu - \cos^2(\phi_1 - \phi_2))} \tag{9.5}$$

$$\ddot{\phi}_2 = \frac{\mu g(\sin\phi_1 \cos(\phi_1 - \phi_2) - \sin\phi_2) + (\mu l_1\dot{\phi}_1^2 + l_2\dot{\phi}_2^2 \cos(\phi_1 - \phi_2))\ \sin(\phi_1 - \phi_2)}{l_2(\mu - \cos^2(\phi_1 - \phi_2))} \tag{9.6}$$

Check all these equations, deriving them from Equations 9.1 and 9.2 in your report.