Logarithms

A **logarithm** is a way to express the relationship between numbers in an exponential expression.

Every operation in mathematics has an opposite. The opposite of adding is subtracting. The opposite of multiplying is dividing. Taking a root (square root, cube root, ...) is one opposite of an exponent. Look at this exponential expression along with its terminology. I can write an expression with 2 and 5 that gives the power, 32, as the answer:

We can rewrite this relationship as a radical expression:

The positions of the numbers have been changed, but it's the same relationship. The radical expression has the 5 and the 32. The base from the original equation, 2, is now the "answer" in the equation.

Neither of these expressions can use the 32 and 2 to give an answer of 5, the exponent from the original equation. That's what logarithms do. When you see a logarithm expression, it asks the question, **"What's the missing exponent?"** Logarithms are another way of writing exponents.

Example 1: Express as a logarithm: $6^3 = 216$ and $\sqrt[7]{x} = y$.

Solution: Logarithms ask the question, "what is the exponent?" This will help us remember where each number goes:

$6^3 = 216$ is equivalent to $\log_6 216 = 3$, and
$\sqrt[7]{x} = y$ is equivalent to $\log_x y = \frac{1}{7}$.

We could also interpret the expressions as $\sqrt[3]{216} = 6$, and so $\log_{6^3} 6 = \frac{1}{3}$ and $y^7 = x$, or $\log_y x = 7$. It's better to just use what you see.

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