8-5: Factoring Special Products

If a polynomial is a **perfect square trinomial**, the polynomial can be factored using a pattern. We discussed these "special cases" near the end of chapter 7.

$$a^2 + 2ab + b^2 = (a + b)^2 \quad \text{or} \quad a^2 - 2ab + b^2 = (a - b)^2$$

Example 1. Determine whether $4x^2 + 20x + 25$ is a perfect square trinomial. If so, factor it. If not, explain why.

Step 1: Find a, b, then 2ab. $a = \sqrt{4x^2} = 2x$ ✓ $b = \sqrt{25} = 5$ ✓ Yes!

Therefore, $2ab = 2 \cdot 2x \cdot 5 = 20x$ ✓

Step 2: Substitute expressions for a and b into $(a + b)^2$

$(2x+5)^2$

Determine whether each trinomial is a perfect square. If so, factor it. If not, explain why.

1. $9y^2 + 20y + 100$
   a = $3y$
   b = 10
   2ab = $2(3y)(10) = 60y$
   Factor or explain: No, the middle term is not 2ab.

2. $x^2 - 14x + 49$
   a = x
   b = 7
   2ab = $2(x)(7) = 14x$
   Factor or explain: $(x-7)^2$

3. $25x^2 + 20x + 4$
   a = $5x$
   b = 2
   2ab = $2(5x)(2) = 20x$
   Factor or explain: $(5x+2)^2$

If a binomial is a **difference of perfect squares**, it can be factored using a pattern:

$$a^2 - b^2 = (a + b)(a - b)$$

Example 2. Determine whether $9x^2 - 25$ is a difference of perfect squares. If so, factor it. If not, explain why.

Step 1: Determine if the binomial is a difference. Is it a subtraction? Yes

Step 2: Find a and b. $a = 3x$ $b = 5$

Therefore,

Step 3: Substitute expressions for a and b into $(a + b)(a - b)$

$(3x+5)(3x-5)$