

$$\text{Let } A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$$

According to the column-row expansion rule:

$$col_1(A)row_1(B) + col_2(A)row_2(B) + col_3(A)row_3(B)$$

Thus

$$col_1(A)row_1(B) = \begin{bmatrix} 1 \\ 4 \end{bmatrix} \times [1 \quad 2] = \begin{bmatrix} 1 & 2 \\ 4 & 8 \end{bmatrix}$$

$$col_2(A)row_2(B) = \begin{bmatrix} 2 \\ 5 \end{bmatrix} \times [3 \quad 4] = \begin{bmatrix} 6 & 8 \\ 15 & 20 \end{bmatrix}$$

$$col_3(A)row_3(B) = \begin{bmatrix} 3 \\ 6 \end{bmatrix} \times [5 \quad 6] = \begin{bmatrix} 15 & 18 \\ 30 & 36 \end{bmatrix}$$

$$\sum_{k=1}^3 col_k(A)row_k(B) = \begin{bmatrix} 1+6+15 & 2+8+18 \\ 4+15+30 & 8+20+36 \end{bmatrix} = \begin{bmatrix} 22 & 28 \\ 49 & 64 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} = \begin{bmatrix} 1+12+15 & 2+8+18 \\ 4+15+30 & 8+20+36 \end{bmatrix} = \begin{bmatrix} 22 & 28 \\ 49 & 64 \end{bmatrix}$$

Both answers are the same and thus prove the column-row expansion.