

**Exponential Functions**

An exponential function grows (or decays) by the same *relative* amount (the same *percentage*) in each unit of time. For a quantity  $Q$  growing exponentially with a fractional growth rate  $r$ ,

$$Q = Q_0 \times (1+r)^t$$

where

$Q$  = value of exponentially growing quantity after time  $t$

$Q_0$  = initial value of the quantity

$r$  = fractional growth rate of the quantity (this is the percentage by which the quantity is growing, converted to decimal).

$t$  = time

Negative values of  $r$  correspond to exponential decay. Note that *the units of time used for  $t$  and  $r$  must be the same*. For example, if the fractional growth rate is 0.05 per month, then  $t$  must also be in months.

*Example 1:*

*The population of Greenville is increasing at a rate of 5.6% per year. If the population today is 8,000, what will it be 10 years from now?*

*Solution:*

Using our formula above with  $Q_0 = 8,000$ ,  $r = .056$  (per year), and  $t = 10$  (years), we have

$$Q = Q_0 \times (1+r)^t$$

$$Q = 8,000 \times (1+.056)^{10}$$

$$Q = 13,795$$

This tells us that 10 years from now the population will be 13,795.