Chapter 4: Linear Systems of Equations

Any linear system of two equations (with two variables) can be solved by using algebra or graphing. To solve systems of equations algebraically, we can use **elimination** or **substitution**. The strategy for both is the same: remove one variable from the system so that you have one equation with one unknown.

METHOD 1 : ELIMINATION

The first step to solving a system of equations using elimination is to make sure both of the equations are in the same form. If they are not, pick either the first or second equation, and rearrange the other equation to match it.

Example: $3x + 5y = 12$
$2y = 8 + 4x$

Solution:

If we pick the first equation, we see the x and y terms are on the left of the equal sign and a number is on the right. To make the second equation match, we would subtract 4x from both sides giving $-4x + 2y = 8$.

The second step is to eliminate one variable and one equation. Here are some tips on how to do this:

(a) If an equation contain decimals or fractions, multiply by a factor that will eliminate the decimals or fractions. This will make it simpler to solve the system.
(b) If the coefficients of one variable are the same number, but opposite signs, the two equations should be added together to eliminate that variable.
(c) If the coefficients of one variable are the same number and the same sign, subtract one equation from the other. (Multiplying one equation by −1 and adding them would have the same effect).
(d) When the variables do not have equal coefficients, an extra step or two is needed. We have to find a factor (or factors) to multiply one (or both) equations by so that one variable has equal coefficients and they can be added or subtracted as in (a) or (b).

After you reach the point of one variable with one equation, solve for that variable. Then plug that value back into one of the two original equations to determine the other variable's value.

Example 1: Solve by elimination:

$$2x + 3y = 13 \quad (1)$$
$$x - 2y = -4 \quad (2)$$

Solution:

The two equations are in the same format, so we can go to step 2.
This set of equations is the fourth case, where neither x nor y has equal coefficients. If we multiply equation #2 by −2, the x terms will have equal number coefficients of opposite signs.

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