MATH 2000 MATRICES - Worksheet Answers

Use notebook paper to work each of the following exercises. You may use a calculator where possible. All answers must be exact and simplified. Write up your problems in your own words. Please use pencil and write neatly on one side of paper only. **BE SURE TO FOLLOW INSTRUCTIONS!**

The following matrices are given:

$$\mathbf{A} = \begin{bmatrix} 3 & -2 & 0 \\ -1 & 0 & 4 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 4 & -8 \\ 6 & -12 \\ 1 & -2 \end{bmatrix} \quad \mathbf{C} = \begin{bmatrix} 5 & -10 \\ -3 & 4 \end{bmatrix} \quad \mathbf{D} = \begin{bmatrix} 5 & 1 & 0 \\ 0 & -2 & 3 \end{bmatrix} \quad \mathbf{E} = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 18 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix}$$

$$\mathbf{F} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \mathbf{G} = \begin{bmatrix} -1 & 2 & 3 \\ 2 & 0 & -2 \\ 3 & -2 & 6 \end{bmatrix} \quad \mathbf{S} = \begin{bmatrix} 0 & 0 & 0 \\ \frac{1}{2} & 1 & \frac{1}{2} \\ 0 & 0 & 0 \end{bmatrix} \quad \mathbf{X} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & j \end{bmatrix} \quad \mathbf{Y} = \begin{bmatrix} k & l & m \\ n & p & r \\ s & t & u \end{bmatrix}$$

1. Find **A** - **D** and -**D** + **A**. What do your results seem to imply about addition of matrices?

   $$\mathbf{A} - \mathbf{D} = \begin{bmatrix} 3 & -2 & 0 \\ -1 & 0 & 4 \end{bmatrix} - \begin{bmatrix} 5 & 1 & 0 \\ 0 & -2 & 3 \end{bmatrix} = \begin{bmatrix} -2 & -3 & 0 \\ -1 & 2 & 1 \end{bmatrix}$$

   $$-\mathbf{D} + \mathbf{A} = -\begin{bmatrix} 5 & 1 & 0 \\ 0 & -2 & 3 \end{bmatrix} + \begin{bmatrix} 3 & -2 & 0 \\ -1 & 0 & 4 \end{bmatrix} = \begin{bmatrix} -2 & -3 & 0 \\ -1 & 2 & 1 \end{bmatrix}$$

   Addition of matrices is commutative.

2. Find **GS** and **SG**. What do your results prove about multiplication of matrices?

   $$\mathbf{GS} = \begin{bmatrix} -1 & 2 & 3 \\ 2 & 0 & -2 \\ 3 & -2 & 6 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ \frac{1}{2} & 1 & \frac{1}{2} \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

   $$\mathbf{SG} = \begin{bmatrix} 0 & 0 & 0 \\ \frac{1}{2} & 1 & \frac{1}{2} \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} -1 & 2 & 3 \\ 2 & 0 & -2 \\ 3 & -2 & 6 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 3 & 0 & 2.5 \\ 0 & 0 & 0 \end{bmatrix}$$

   Multiplication of matrices is not commutative.

3. Find **CF** and **FC**. Why are these products equal? What real number is analogous to **F**?

   $$\mathbf{CF} = \begin{bmatrix} 5 & -10 \\ -3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 5 & -10 \\ -3 & 4 \end{bmatrix} \text{ and } \mathbf{FC} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 5 & -10 \\ -3 & 4 \end{bmatrix} = \begin{bmatrix} 5 & -10 \\ -3 & 4 \end{bmatrix}$$

   F is the 3x3 multiplicative identity. The real number equivalent of I_3 is 1.

4. Why can't you find **AD**? Explain the rule for determining if a matrix product exists.

   The number of columns of **A** is not equal to the number of rows of **D**.