

Multiplication from left distributes over addition:

i.e., If A, B and C are three matrices.

$$A(B + C) = AB + BC.$$

Example:

$$A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}; B = \begin{bmatrix} 4 & 2 \\ 1 & 0 \end{bmatrix}; C = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

Show that $A(B + C) = AB + BC$.

Solution:

$$B + C = \begin{bmatrix} 4 & 2 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 4+1 & 2+2 \\ 1+3 & 0+4 \end{bmatrix} \Rightarrow \begin{bmatrix} 5 & 4 \\ 4 & 4 \end{bmatrix}$$

$$A(B + C) = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 5 & 4 \\ 4 & 4 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 \times 5 + 3 \times 4 & 1 \times 4 + 3 \times 4 \\ 2 \times 5 + 4 \times 4 & 2 \times 4 + 4 \times 4 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 5+12 & 4+12 \\ 10+16 & 8+16 \end{bmatrix} \Rightarrow \begin{bmatrix} 17 & 16 \\ 26 & 24 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 4 & 2 \\ 1 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 \times 4 + 3 \times 1 & 1 \times 2 + 3 \times 0 \\ 2 \times 4 + 4 \times 1 & 2 \times 2 + 4 \times 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4+3 & 2+0 \\ 8+4 & 4+0 \end{bmatrix} \Rightarrow \begin{bmatrix} 7 & 2 \\ 12 & 4 \end{bmatrix}$$