Math 141: College Algebra
Worksheet 10

1. Express the following in exponential form:
   a) $\log_2 64 = 6$ $\quad 2^6 = 64$
   b) $\log_2 \frac{1}{16} = -4$ $\quad 2^{-4} = \frac{1}{16}$
   c) $\log_8 4 = \frac{2}{3}$ $\quad 8^{2/3} = 4$
   d) $\ln 20.09 \approx 3$ $\quad e^3 \approx 20.09$

2. Express each of the following in logarithmic form:
   a) $2^3 = 8$ $\quad \log_2 8 = 3$
   b) $81^{1/2} = 9$ $\quad \log_{81} 9 = \frac{1}{2}$
   c) $e^{2/3} \approx 1.95$ $\quad \ln 1.95 \approx \frac{2}{3}$
   d) $(\frac{1}{8})^3 = \frac{1}{512}$ $\quad \log_{\frac{1}{8}} \frac{1}{512} = 3$

3. Calculate
   a) $\log_2 32 = 5$ $\quad (2^5 = 32)$
   b) $\log_{27} 3 = \frac{1}{3}$ $\quad (27^{1/3} = 3)$
   c) $\log_e 1 = 0$ $\quad$ (log to any base of 1 is zero)
   d) $\log_\pi \pi^2 = 2$ $\quad (\pi^2 = \pi^2)$
   e) $\log_{\sqrt{3}} \frac{1}{9} = -4$ $\quad (\sqrt{3}^{-4} = 3^{-2} = \frac{1}{9})$

4. Express $\log_{1000} 100$ in terms of $\log_{10}$, and then calculate the value. We can use

$$\log_b x = \log_a x / \log_a b$$

   which gives

$$\log_{1000} 100 = \log_{10} 100 / \log_{10} 1000 = \log_{10} 10^2 / \log_{10} 10^3 = \frac{2}{3}$$

5. Solve: $\ln(\log_{10} x) = 0$.
   We start by raising both sides to the power of e to get rid of ln:

$$\begin{aligned} \ln(\log_{10} x) &= 0 \\ e^{\ln(\log_{10} x)} &= e^0 \\ \log_{10} x &= 1 \end{aligned}$$

   where we used that $e^{\ln a} = a$. Next we raise both sides to the power of 10 to get rid of $\log_{10}$:

$$\begin{aligned} \log_{10} x &= 1 \\ 10^{\log_{10} x} &= 10^1 \\ x &= 10 \end{aligned}$$

   where we used that $10^{\log_{10} a} = a$.

6. Population Doubling Times.

   (a) Imagine that a population grows exponentially with a rate k. If the initial population is P_0, write down an equation relating the final population P_f and the elapsed time T.

$$P_f(T) = P_0 e^{kT}$$