VANCOUVER COMMUNITY COLLEGE

Maxima & Minima

Aside from finding the tangent line to a curve, derivatives can also be used to find the highest or lowest value for a function given a set of constraints. For a curve that is continuous over the entire domain of a function, these extreme points, or **extrema** (pl. of extremum), can only occur where the derivative is equal to zero. The **critical values** for a function are those x-values where the derivative of the function is zero, or the derivative does not exist.

If an extreme point is the highest point in its neighbourhood, it's called a **maximum**, and if it's the lowest, it's a **minimum**. If there are other values for *f(x)* elsewhere that surpass these points, then these are called a **local maximum** or a **local minimum**. If a value for *f(x)* is the highest or lowest possible for the function, then it is a **global maximum** or a **global minimum**.

Example 1: Find all extreme points on the curve $y = x^4 - 4x^3 - 20x^2 + 5$.

Solution: First, we take the derivative, then we find its zeroes:

$$\begin{aligned} y &= x^4 - 4x^3 - 20x^2 + 5 \\ y' &= 4x^3 - 12x^2 - 40x \\ &= 4x(x^2 - 3x - 10) \\ &= 4x(x - 5)(x + 2) \\ 0 &= 4x(x - 5)(x + 2) \\ x &= -2, 0 \text{ or } 5 \end{aligned}$$

We plug these values for x into our original equation for y:

$$\begin{aligned} y &= [-2]^4 - 4[-2]^3 - 20[-2]^2 + 5 \\ &= -27 \\ y &= [0]^4 - 4[0]^3 - 20[0]^2 + 5 \\ &= 5 \\ y &= [5]^4 - 4[5]^3 - 20[5]^2 + 5 \\ &= -370 \end{aligned}$$

The critical points are (−2, −27), (0, 5) and (5, −370). We can tell whether these are maxima or minima by applying the Second Derivative Test. If the result is positive, then the point is a minimum. If the result is negative, the point is a maximum. (We'll look at the possibility that the result is zero on the next page.)

$$\begin{aligned} y' &= 4x^3 - 12x^2 - 40x \\ y'' &= 12x^2 - 24x - 40 \end{aligned}$$

when $x = -2$
$y'' = 12[-2]^2 - 24[-2] - 48$
$= 56$ minimum!

when $x = 0$
$y'' = 12[0]^2 - 24[0] - 40$
$= -40$ maximum!

when $x = 5$
$y'' = 12[5]^2 - 24[5] - 40$
$= 140$ minimum!

Authored by Darren Rigby