MA 12 LG 15 (Combinatorics)

1.1 The Fundamental Counting Principle

If all combinations = a × b × c × ...

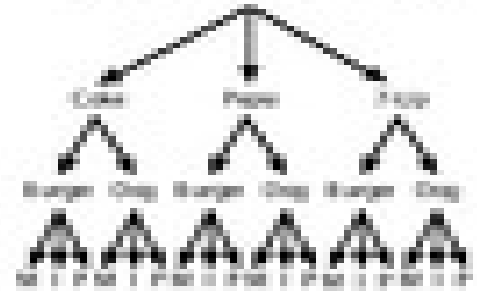
where a, b, c, are the number of choices in each category

Example for 1.1

Draw a tree diagram to list all the possibilities for the following:

How many different combinations can be made from:

Drinks	Dinners	Desserts
Coke	Burger	Ice cream
Pepsi	Hot Dog	Ice Cream Pie
7-Up		



# of Combos = 3 × 2 × 3 = 18 combos

1.2 Permutations

Factorial: $n! = n(n-1)(n-2)(n-3)\ldots(n-(n-1))$

Permutations (order is important) also is different from with

$$\text{Permutations} = {}_nP_r = \frac{n!}{(n-r)!}$$ n = # of objects

a, b, c, ... = # of objects alike

$$nP_n = \frac{n!}{(n-n)!}$$

Example for 1.2

Express in factorial notation:

8 × 7 × 6 × 5 × 4 × 3 × 2 × 1 = 8!

Example for 1.21

Express as a product of natural #'s in reduced form:

$$\frac{12}{8} = \frac{3 \times 4}{2 \times 4}$$

$$\frac{12P_3}{8P_3} = \frac{\frac{12!}{(12-3)!}}{\frac{8!}{(8-3)!}} = \frac{12!}{8!} = \frac{12 \times 11 \times 10 \times 9!}{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}$$

$$= \frac{12 \times 11 \times 10}{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}$$

$$= \frac{3 \times 4 \times 11 \times 2 \times 5 \times 3 \times 2}{2 \times 4 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}$$

$$= \frac{11}{7 \times 6 \times 4}$$

Example for 1.22

Express $\frac{12 \times 11 \times 10 \times 9 \times 8}{4 \times 3 \times 2 \times 1}$ in the form $\frac{n!}{(n-r)!}$

$$\frac{12 \times 11 \times 10 \times 9 \times 8}{4 \times 3 \times 2 \times 1} = \frac{\frac{12!}{(12-5)!}}{5!} = \frac{12!}{5!7!} = {}_{12}C_5$$

n = 12 r = 5

1.3 Combinations

Factorial: $n! = n(n-1)(n-2)(n-3)\ldots(n-(n-1))$

$$\text{Permutations} = {}_nP_r = \frac{n!}{(n-r)!}$$

Combinations (order isn't important) also is same as with

$${}_nC_r = \frac{{}_nP_r}{r!} = \frac{n!}{r!(n-r)!}$$

Example for 1.31

Express as a product in reduced form:

$${}_{12}C_5 = \frac{12!}{(12-5)!5!} = \frac{12!}{7!5!} = \frac{12 \times 11 \times 10 \times 9 \times 8 \times 7!}{7!5 \times 4 \times 3 \times 2 \times 1} = \frac{12 \times 11 \times 10 \times 9 \times 8}{5 \times 4 \times 3 \times 2 \times 1} = 792$$

$${}_{12}C_7 = \frac{12!}{(12-7)!7!} = \frac{12!}{5!7!} = \frac{12 \times 11 \times 10 \times 9 \times 8 \times 7!}{5 \times 4 \times 3 \times 2 \times 1 \times 7!} = \frac{12 \times 11 \times 10 \times 9 \times 8}{5 \times 4 \times 3 \times 2 \times 1} = 792$$