

# MA 12 LG 15 (Combinatorics)

## 1.1. The Fundamental Counting Principle

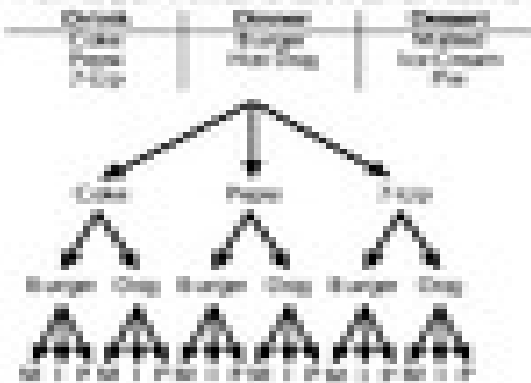
If  $A$  and  $B$  are disjoint sets, then

where  $a, b, c$ , and  $d$  are the number of items in each category.

### Example for 1.1

Draw a tree diagram to find all the possibilities for the following:

How many different combinations can be made from



Total possibilities =  $3 \times 2 \times 4 = 24$  possibilities

## 1.2. Permutations

General:  $n! = n(n-1)(n-2)\dots(2)(1) = n \times (n-1) \dots 1$

Permutations (order is important) are different from sets

Permutations =  $\frac{n!}{(n-r)!}$        $n$  = total # objects  
 $r$  = # to select       $r$  = # to select       $n-r$  = # of objects after

$${}^n P_r = \frac{n!}{(n-r)!}$$

### Example for 1.2

3 objects in factorial notation:

the  $3!$  is  $3 \times 2 \times 1 = 6$

### Example for 1.1

3 objects in product of factorial notation:

$3! = 3 \times 2 \times 1 = 6$

$${}^3 P_3 = \frac{3!}{(3-3)!} = \frac{3!}{0!} = \frac{3!}{1} = 3! = 3 \times 2 \times 1 = 6$$

### Example for 1.2

3 objects, 2 in 1st and 2nd, 1 in 3rd, factorial notation:

$${}^3 P_2 = \frac{3!}{(3-2)!} = \frac{3!}{1!} = \frac{3!}{1} = 3! = 3 \times 2 \times 1 = 6$$

2 in 1st and 2nd, 1 in 3rd:

$${}^3 P_1 = \frac{3!}{(3-1)!} = \frac{3!}{2!} = \frac{3!}{2 \times 1} = 3$$

1 in 1st, 2 in 2nd:

$${}^3 P_2 = \frac{3!}{(3-2)!} = \frac{3!}{1!} = \frac{3!}{1} = 3! = 3 \times 2 \times 1 = 6$$

## 1.3. Combinations

General:  $n! = n(n-1)(n-2)\dots(2)(1) = n \times (n-1) \dots 1$

$${}^n C_r = \frac{n!}{r!(n-r)!}$$

Combinations (order not important) are different from sets

$${}^n C_r = \frac{n!}{r!(n-r)!}$$

### Example for 1.3

3 objects in product of factorial notation:

$${}^3 C_3 = \frac{3!}{3!(3-3)!} = \frac{3!}{3! \cdot 0!} = \frac{3!}{3! \cdot 1} = \frac{3!}{3!} = 1$$

the  $3!$  is  $3 \times 2 \times 1 = 6$

$${}^3 C_2 = \frac{3!}{2!(3-2)!} = \frac{3!}{2! \cdot 1!} = \frac{3!}{2! \cdot 1} = \frac{3!}{2} = \frac{3 \times 2 \times 1}{2} = 3$$

the  $3!$  is  $3 \times 2 \times 1 = 6$