

MA 12 LG 15 (Combinatorics)

1.1. The Fundamental Counting Principle

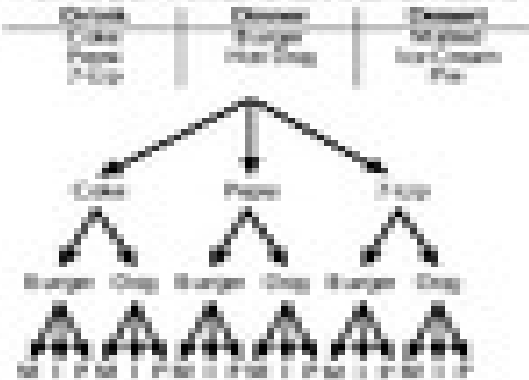
If P and Q are disjoint sets, then

where $n(P)$ and $n(Q)$ are the number of items in each category.

Example for 1.1

Draw a tree diagram to list all the possibilities for the following:

How many different combinations can be made from



Total possibilities = $3 \times 2 \times 2 = 12$ combinations

1.2. Permutations

General: $n! = n(n-1)(n-2)\dots(2)(1) = n \times (n-1) \dots 1$

Permutations (order is important) are different from sets

Permutations = $\frac{n!}{n!}$ = total of objects
 $n! / k!$ = Per objects after

$${}^n P_r = \frac{n!}{(n-r)!}$$

Example for 1.2

3 objects in factorial notation:
 the 3rd factorial is $3! = 2 \times 3 \times 1 = 6$

Example for 1.1

3 objects in product of factorial notation:
 $3! = 3 \times 2 \times 1 = 6$

$${}^3 P_3 = \frac{3!}{(3-3)!} = \frac{3!}{0!} = \frac{3 \times 2 \times 1}{1} = 6$$

Example for 1.2

3 objects, 2 in 1st and 2nd, 1 in 3rd, factorial notation:

$${}^3 P_2 = \frac{3!}{(3-2)!} = \frac{3!}{1!} = \frac{3 \times 2 \times 1}{1} = 6$$

$${}^3 P_1 = \frac{3!}{(3-1)!} = \frac{3!}{2!} = \frac{3 \times 2 \times 1}{2 \times 1} = 3$$

$${}^3 P_0 = \frac{3!}{(3-0)!} = \frac{3!}{3!} = \frac{3 \times 2 \times 1}{3 \times 2 \times 1} = 1$$

1.3. Combinations

General: $n! = n(n-1)(n-2)\dots(2)(1) = n \times (n-1) \dots 1$

$${}^n C_r = \frac{n!}{r!(n-r)!}$$

Combinations (order not important) are different from sets

$${}^n C_r = \frac{n!}{r!(n-r)!} = \frac{n!}{r!n!} = \frac{1}{r!}$$

Example for 1.3

3 objects in product of factorial notation:

$${}^3 C_3 = \frac{3!}{3!(3-3)!} = \frac{3!}{3!0!} = \frac{3 \times 2 \times 1}{3 \times 2 \times 1 \times 1} = 1$$

= the 3rd factorial

$${}^3 C_2 = \frac{3!}{2!(3-2)!} = \frac{3!}{2!1!} = \frac{3 \times 2 \times 1}{2 \times 1 \times 1} = 3$$

= the 3rd factorial

$${}^3 C_1 = \frac{3!}{1!(3-1)!} = \frac{3!}{1!2!} = \frac{3 \times 2 \times 1}{1 \times 2 \times 1} = 3$$

= the 3rd factorial

$${}^3 C_0 = \frac{3!}{0!(3-0)!} = \frac{3!}{0!3!} = \frac{3 \times 2 \times 1}{1 \times 3 \times 2 \times 1} = 1$$

= 1