MA 12 LG 15 (Combinatorics)

1.1 The Fundamental Counting Principle

of combinations = a x b x c x ...

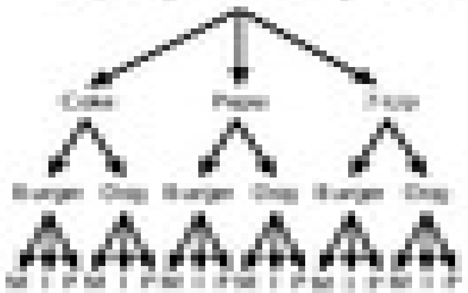
where a, b, c, are the number of choices in each category

Example for 1.4

Draw a tree diagram to list all the possibilities for the following.

How many different combinations can be made from:

Drink	Dinner	Dessert
Coke	Burger	Sundae
Pepsi	Hot Dog	Ice Cream
7-Up		Pie



# of combos = 3 x 2 x 3 = 18 combos

1.2 Permutations

Factorial $n! = n(n-1)(n-2)(n-3) \ldots x1$

Permutations: (order is important) abc is different from acb

Permutations $= \frac{n!}{a!b!c!\ldots}$ n = # of objects

a, b, c, ... = # of objects alike

$_nP_r = \frac{n!}{(n-r)!}$

Example for 1.4

Express in factorial notation.

8 x 7 x 6 x 5 x 4 x 3 x 2 x 1 = 8!

Example for 1.13

Express as a product of natural #'s in reduced form.

5! = 5 x 4 x 3 x 2 x 1

$$_{10}P_7 = \frac{10!}{(10-7)!} = \frac{10!}{3!}$$

$$= \frac{10x9x8x7x6x5x4x3x2x1}{3x2x1}$$

$$= \mathbf{10x9x8x7x6x5x4}$$

Example for 10.20

Express 12 x 11 x 10 x 9 in the form $_nP_r$

$$_nP_r = \frac{n!}{(n-r)!}$$

$$12 \times 11 \times 10 \times 9 = \frac{12x11x10x9x8x7x6x5x4x3x2x1}{8x7x6x5x4x3x2x1}$$

$$= \frac{12!}{8!} = \frac{12!}{(12-8)!}$$

n = 12 r = 4 $= {}_{12}P_4$

1.3 Combinations

Factorial $n! = n(n-1)(n-2)(n-3) \ldots x1$

$_nP_r = \frac{n!}{(n-r)!}$

Combinations: (order not important) abc is same as acb

$$_nC_r = \frac{_nP_r}{r!} = \frac{n!}{r!(n-r)!}$$

Example for 1.6

Express as a product in reduced form.

$$_{10}P_5 = \frac{10!}{(10-5)!} = \frac{10!}{5!}$$

$$= \frac{10x9x8x7x6x5x4x3x2x1}{5x4x3x2x1}$$

$$= 10 x 9 x 8 x 7 x 6$$

$$= \mathbf{30240}$$

$$_{10}C_5 = \frac{10!}{(10-5)!5!} = \frac{10!}{5!5!}$$

$$= \frac{10x9x8x7x6x5x4x3x2x1}{5x4x3x2x1x5x4x3x2x1}$$

$$= \frac{10x9x8x7x6}{5x4x3x2x1}$$

$$= 9 x 7 x 2 x 2$$

$$= \mathbf{252}$$