

MA 12 LG 15 (Combinatorics)

1.1. The Fundamental Counting Principle

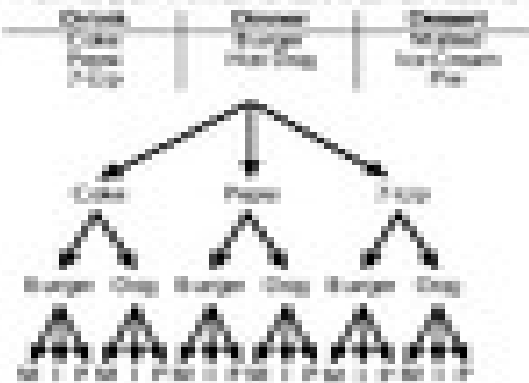
If P and Q are disjoint events, then

where A, B, C are the number of items in each category.

Example for 1.1

Draw a tree diagram to find all the possibilities for the following:

How many different combinations can be made from



Total possibilities = $3 \times 2 \times 2 = 12$ combinations

1.2. Permutations

General: $n! = n(n-1)(n-2)\dots(2)(1) = n \times (n-1) \dots 1$

Permutations (order is important) are different from sets

Permutations = $\frac{n!}{(n-r)!}$ n = total # objects
 r = # to select r = # of objects after

$${}^n P_r = \frac{n!}{(n-r)!}$$

Example for 1.2

3 objects in factorial notation:

the $3!$ is $3 \times 2 \times 1 = 6 = 3 \times 2 \times 1 = 6$

Example for 1.1

3 objects in a product of factorial (P) or combination form:

$3! = 3 \times 2 \times 1 = 6$

$$\begin{aligned} {}^3 P_3 &= \frac{3!}{(3-3)!} = \frac{3!}{0!} \\ &= \frac{3 \times 2 \times 1}{1} = 6 \\ &= 6 \end{aligned}$$

Example for 1.2

3 objects: $3!$ is $3 \times 2 \times 1 = 6$ but the form ${}^3 P_2$

${}^3 P_2 = \frac{3!}{(3-2)!}$

$3!$ is $3 \times 2 \times 1 = 6$ $(3-2)!$ is $1 \times 1 = 1$

$$\begin{aligned} {}^3 P_2 &= \frac{3!}{(3-2)!} = \frac{3!}{1!} \\ &= \frac{3 \times 2 \times 1}{1} = 6 \end{aligned}$$

1.3. Combinations

General: $n! = n(n-1)(n-2)\dots(2)(1) = n \times (n-1) \dots 1$

${}^n C_r = \frac{n!}{r!(n-r)!}$

Combinations (order not important) are different from sets

${}^n C_r = \frac{n!}{r!(n-r)!}$

Example for 1.3

3 objects in product or factorial form:

$$\begin{aligned} {}^3 C_3 &= \frac{3!}{3!(3-3)!} = \frac{3!}{3! \times 0!} \\ &= \frac{3 \times 2 \times 1}{(3 \times 2 \times 1) \times 1} \\ &= \frac{6}{6 \times 1} \\ &= 1 \end{aligned}$$

$$\begin{aligned} {}^3 C_2 &= \frac{3!}{2!(3-2)!} = \frac{3!}{2! \times 1!} \\ &= \frac{3 \times 2 \times 1}{(2 \times 1) \times 1} \\ &= \frac{6}{2 \times 1} \\ &= 3 \end{aligned}$$