

# MA 12 LG 15 (Combinatorics)

## 1.1. The Fundamental Counting Principle

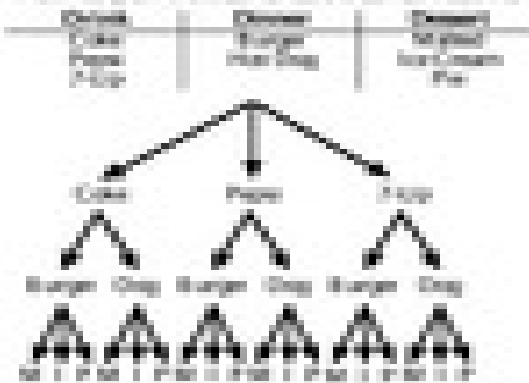
If  $P$  and  $Q$  are disjoint sets, then

where  $n(P)$ ,  $n(Q)$  are the number of items in each category.

### Example for 1.1

Draw a tree diagram to list all the possibilities for the following:

How many different combinations can be made from



Total possibilities =  $3 \times 2 \times 2 = 12$  combinations

## 1.2. Permutations

General:  $n! = n(n-1)(n-2)\dots(2)(1) = n \times (n-1) \dots 1$

Permutations (order is important) are different from sets

Permutations =  $\frac{n!}{(n-r)!}$      $n$  = total # objects  
 $r$  = # to select     $r$  = # of objects after

$${}^n P_r = \frac{n!}{(n-r)!}$$

### Example for 1.2

3 objects in factorial notation:

the  $3!$  =  $3 \times 2 \times 1 = 6 = 3 \times 2 \times 1 = 6$

### Example for 1.1

3 objects in product of factorial  $n!$  or factorial notation:

$$3! = 3 \times 2 \times 1 = 6$$

$${}^3 P_3 = \frac{3!}{(3-3)!} = \frac{3!}{0!} = \frac{3!}{1} = 3! = 3 \times 2 \times 1 = 6$$

### Example for 1.2

3 objects, 2 in 1st & 2 in 2nd =  ${}^3 P_2$

$${}^3 P_2 = \frac{3!}{(3-2)!} = \frac{3!}{1!} = \frac{3 \times 2 \times 1}{1} = 3 \times 2 = 6$$

2 in 1st & 1 in 2nd =  ${}^3 P_1$

$${}^3 P_1 = \frac{3!}{(3-1)!} = \frac{3!}{2!} = \frac{3 \times 2 \times 1}{2 \times 1} = \frac{3 \times 2}{2} = 3$$

1 in 1st, 1 in 2nd =  ${}^3 P_0$

$${}^3 P_0 = \frac{3!}{(3-0)!} = \frac{3!}{3!} = \frac{3 \times 2 \times 1}{3 \times 2 \times 1} = 1$$

## 1.3. Combinations

General:  $n! = n(n-1)(n-2)\dots(2)(1) = n \times (n-1) \dots 1$

$${}^n C_r = \frac{n!}{r!(n-r)!}$$

Combinations (order not important) are different from sets

$${}^n C_r = \frac{n!}{r!(n-r)!} = \frac{n!}{r!n!} = \frac{1}{r!}$$

### Example for 1.3

3 objects in product or factorial notation:

$${}^3 C_3 = \frac{3!}{3!(3-3)!} = \frac{3!}{3!0!} = \frac{3!}{3!1} = \frac{3!}{3!} = 1$$

the  $3!$  =  $3 \times 2 \times 1 = 6 = 3 \times 2 \times 1 = 6$

$${}^3 C_2 = \frac{3!}{2!(3-2)!} = \frac{3!}{2!1!} = \frac{3!}{2!1} = \frac{3 \times 2 \times 1}{2 \times 1} = \frac{3 \times 2}{2} = 3$$

the  $3!$  =  $3 \times 2 \times 1 = 6 = 3 \times 2 \times 1 = 6$