MA 12 LG 15 (Combinatorics)

1.1 The Fundamental Counting Principle

of combinations = $a \times b \times c \times \ldots$

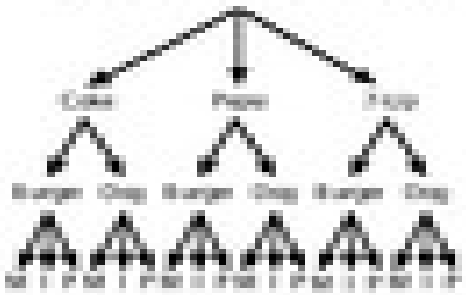
where $a, b, c, \ldots$ are the number of choices in each category

Example for 1.1

Draw a tree diagram to list all the possibilities for the following:

How many different combinations can be made from:

Drink	Dessert	Dessert
Coke	Burger	Apple
Pepsi	Hot Dog	Ice Cream
7-Up		Pie



# of combos = $3 \times 2 \times 3$ = 18 combos

1.2 Permutations

Factorial $n! = n(n-1)(n-2)(n-3)\ldots(1)$

Permutations (order is important) are to different from sets

Permutations = $\frac{n!}{(n-r)!}$ n = # of objects

$r, s, t, \ldots$ = # of objects alike

$_nP_r = \frac{n!}{(n-r)!}$

Example for 1.2

Express in factorial notation:

$8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 8!$

Example for 1.3

Express as a product of natural #'s in reduced form:

$9! = 9 \times 8 \times 7 \times 6 \times 5 \times 4$

$_9P_4 = \frac{9!}{(9-4)!} = \frac{9!}{5!}$

$= \frac{(9)(8)(7)(6)(5)(4)(3)(2)(1)}{(5)(4)(3)(2)(1)}$

$= \frac{9 \times 8 \times 7 \times 6}{1}$

$= 9 \times 8 \times 7 \times 6$

Example for 1.4

Express $12 \times 11 \times 10 \times 9 \times 8$ in the form $_nP_r$

$_nP_r = \frac{n!}{(n-r)!}$

$12 \times 11 \times 10 \times 9 \times 8 = \frac{(12)(11)(10)(9)(8)(7)(6)(5)(4)(3)(2)(1)}{(7)(6)(5)(4)(3)(2)(1)}$

$= \frac{12!}{7!} = \frac{12!}{(12-7)!}$

$n = 12 \quad r = 5 \qquad = {}_{12}P_5$

1.3 Combinations

Factorial $n! = n(n-1)(n-2)(n-3)\ldots(1)$

$_nC_r = \frac{n!}{(n-r)!}$

Combinations (order isn't important) are to sets as

$_nC_r = \frac{_nP_r}{r!} = \frac{n!}{(n-r)!}$

Example for 1.5

Express as a product in reduced form:

$_{10}P_5 = \frac{10!}{(10-5)!} = \frac{10!}{5!}$

$= \frac{(10)(9)(8)(7)(6)(5)(4)(3)(2)(1)}{(5)(4)(3)(2)(1)}$

$= (10) \times 9 \times 8 \times 7 \times 6$

$= 30240$

$_{10}C_5 = \frac{10!}{(10-5)!5!} = \frac{10!}{5!5!}$

$= \frac{(10)(9)(8)(7)(6)(5)(4)(3)(2)(1)}{(5)(4)(3)(2)(1)(5)(4)(3)(2)(1)}$

$= \frac{9 \times 8 \times 7 \times 6}{4 \times 3 \times 2 \times 1}$

$= \frac{9 \times 7 \times 2 \times 3}{3 \times 2}$

$= 252$