

use the Factor Theorem to find factors of the function

$$f(x) = x^3 + 3x^2 - x - 3$$

choosing factors of the highest constant 3

1, -1, 3, -3

$$\begin{aligned} f(1) &= (1)^3 + 3(1)^2 - (1) - 3 \\ &= 1 + 3 - 1 - 3 = 0 \quad \therefore \underline{(x - 1) \text{ a factor}} \end{aligned}$$

$$\begin{aligned} f(-1) &= (-1)^3 + 3(-1)^2 - (-1) - 3 \\ &= -1 + 3 + 1 - 3 = 0 \quad \therefore \underline{(x + 1) \text{ a factor}} \end{aligned}$$

$$\begin{aligned} f(3) &= (3)^3 + 3(3)^2 - (3) - 3 \\ &= 27 + 27 - 3 - 3 = 48 \quad \therefore \underline{(x - 3) \text{ not a factor}} \end{aligned}$$

$$\begin{aligned} f(-3) &= (-3)^3 + 3(-3)^2 - (-3) - 3 \\ &= -27 + 27 + 3 - 3 = 0 \quad \therefore \underline{(x + 3) \text{ a factor}} \end{aligned}$$

$$\therefore \underline{x^3 + 3x^2 - x - 3 = (x - 1)(x + 1)(x + 3)}$$

n.b. the sign change of the constant $f(5) \Rightarrow (x - 5)$