

**EXCELSIOR EDUCATION**  
Department of Mathematics

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Prove the following trigonometric identities.

1.  $\sec \theta - \tan \theta \sin \theta = \cos \theta$
2.  $\cot^2 \alpha (1 - \cos^2 \alpha) = \cos^2 \alpha$
3.  $\frac{1 + \sec \beta}{\sin \beta + \tan \beta} = \csc \beta$
4.  $\frac{1 + \sin \varphi}{\cos \varphi} = \frac{\cos \varphi}{1 - \sin \varphi}$
5.  $\sin^2 t - \cos^2 t = 1 - 2 \cos^2 t$
6.  $\frac{\tan \gamma}{\sec \gamma - 1} = \frac{\sec \gamma + 1}{\tan \gamma}$
7.  $\frac{\sin \theta - \sin \theta \cos \theta}{\sin \theta + \sin \theta \tan \theta} = \frac{1 - \cos \theta}{1 + \tan \theta}$
8.  $\frac{\cos^2 \alpha}{1 - \sin \alpha} = 1 + \sin \alpha$
9.  $\sec^2 t \csc^2 t = \sec^2 t + \csc^2 t$
10.  $\frac{\sec x - \csc x}{\sec x + \csc x} = \frac{\tan x - 1}{\tan x + 1}$
11.  $1 - \frac{\cos^2 \beta}{1 + \sin \beta} = \sin \beta$
12.  $\frac{\sin \varphi + \tan \varphi}{\cot \varphi + \csc \varphi} = \sin \varphi \tan \varphi$
13.  $\frac{\sin \alpha \cos \alpha}{\cos^2 \alpha - \sin^2 \alpha} = \frac{\tan \alpha}{1 - \tan^2 \alpha}$
14.  $\frac{\sin \theta - \cos \theta}{\cos \theta} + 1 = \tan \theta$
15.  $\frac{1}{\tan \theta} + \tan \theta = \sec \theta \csc \theta$
16.  $\frac{1 + \cos \omega}{1 - \cos \omega} = \frac{\sec \omega + 1}{\sec \omega - 1}$
17.  $\frac{\sin \omega + \tan \omega}{\sin \omega} = 1 + \sec \omega$
18.  $\frac{\sec \beta + \csc \beta}{1 + \tan \beta} = \csc \beta$
19.  $(1 - \sin \theta)(\sec \theta + \tan \theta) = \cos \theta$
20.  $(\sec \gamma - \tan \gamma)^2 = \frac{1 - \sin \gamma}{1 + \sin \gamma}$
21.  $\sin^2 t - \cos^2 t = 2 \sin^2 t - 1$
22.  $\frac{\cot \theta + \tan \theta}{\sin \theta \cos \theta} = \csc^2 \theta \sec^2 \theta$
23.  $\frac{\csc \alpha}{1 - \csc^2 \alpha} = -\sec \alpha \tan \alpha$
24.  $\frac{\tan \varphi}{1 + \tan^2 \varphi} = \sin \varphi \cos \varphi$
25.  $\frac{\tan^2 \theta}{\sec \theta} = \sec \theta - \cos \theta$
26.  $\cos t \csc t + \tan t = \sec t \csc t$
27.  $\frac{\cos \alpha}{1 + \sin \alpha} + \frac{1 + \sin \alpha}{\cos \alpha} = 2 \sec \alpha$
28.  $\frac{\tan \theta - \cot \theta}{\tan \theta + \cot \theta} = \frac{\tan^2 \theta - 1}{\sec^2 \theta}$
29.  $\frac{\cot \beta}{1 + \cot^2 \beta} = \sin \beta \cos \beta$
30.  $\frac{\tan \varphi + 1}{\sin \varphi + \cos \varphi} = \sec \varphi$
31.  $\cos^2 \psi - \sin^2 \psi = \frac{1 - \tan^2 \psi}{1 + \tan^2 \psi}$
32.  $\frac{\sin \mu}{1 + \cos \mu} = \csc \mu - \cot \mu$
33.  $\cot^2 \theta - \cos^2 \theta = \cos^2 \theta \cot^2 \theta$
34.  $\cos^4 \varphi - \sin^4 \varphi = \cos^2 \varphi - \sin^2 \varphi$
35.  $\sin^3 \alpha + \cos^3 \alpha = (\sin \alpha + \cos \alpha)(1 - \sin \alpha \cos \alpha)$
36.  $(\tan \varphi + \cot \varphi)^2 = \sec^2 \varphi + \csc^2 \varphi$
37.  $(\sin \beta + \cos \beta)(\sec \beta - \csc \beta) = \tan \beta - \cot \beta$
38.  $\frac{\sin \varphi}{1 - \cos \varphi} - \cot \varphi = \csc \varphi$
39.  $\frac{\tan \mu \sin \mu}{\tan \mu + \sin \mu} = \frac{\tan \mu - \sin \mu}{\tan \mu \sin \mu}$
40.  $\frac{\sec \theta + \csc \theta}{\tan \theta + \cot \theta} = \sin \theta + \cos \theta$
41.  $\csc^2 t - \sec^2 t = \cot^2 t - \tan^2 t$
42.  $\frac{\sin^3 \alpha + \csc^3 \alpha}{\sin \alpha + \csc \alpha} = \csc^2 \alpha - \cos^2 \alpha$
43.  $\frac{\cos \varphi \sin^2 \varphi}{1 + \cos \varphi} = \cos \varphi - \cos^2 \varphi$
44.  $\frac{\sin \beta}{1 + \cos \beta} + \frac{1 + \cos \beta}{\sin \beta} = 2 \cot \beta \sec \beta$
45.  $\frac{\sec \theta}{\cot \theta + \tan \theta} = \sin \theta$
46.  $\sin^2 t + \sin^2 t \tan^2 t = \tan^2 t$