

8.1 Rational Expressions and Their Simplification

A rational number is a fraction, like  $\frac{2}{5}$ . A **rational expression** is a ratio in which the numerator and denominator may both be polynomials, like  $\frac{2x+3}{2x^2+5x+3}$ . We need to be careful about having variables in the denominator. Certain values for the variable will cause division by zero. In this case the expression is undefined.

EXAMPLE: Find all numbers for which each rational expression is undefined.

$$\frac{2x+3}{2x^2+5x+3} = \frac{2x+3}{(2x+3)(x+1)}$$

The expression is undefined if the denominator is zero. We have two factors that may cause this:

$$\begin{aligned} (2x+3)(x+1) &= 0 \\ 2x+3 &= 0 \quad \text{or} \quad x+1 = 0 \\ x &= -\frac{3}{2} \quad \text{or} \quad x = -1 \end{aligned}$$

EXAMPLE: Find all numbers for which each rational expression is undefined.

a.)  $\frac{2}{x-4}$       b.)  $\frac{3x+9}{x+5}$       c.)  $\frac{14x^3 - x^2 + 9x - 2}{x^2 - 16}$       d.)  $\frac{3x}{6x^2 - x - 2}$       e.)  $\frac{5}{x^2 + 4}$

Notice that it is the denominator that we are worried about. The numerator doesn't even have to be considered.

**SIMPLIFYING RATIONAL EXPRESSIONS:**

A rational expression can be simplified by dividing out common factors shared by its numerator and denominator. For example,

$$\frac{585}{273} = \frac{3 \cdot 3 \cdot 5 \cdot 13}{3 \cdot 7 \cdot 13} = \frac{3 \cdot 5}{7} = \frac{15}{7}$$

We can't cancel common terms:

WRONG  $\frac{3x+9}{3} = \frac{\cancel{3}x+9}{\cancel{3}} \neq \frac{1x+9}{1} = x+9$

We can cancel common factors:

RIGHT  $\frac{3x+9}{3} = \frac{\cancel{3}(x+3)}{\cancel{3}} = x+3$

TO SIMPLIFY A RATIONAL EXPRESSION  
 1.) Factor the numerator and denominator completely  
 2.) Divide out all the common factors

EXAMPLE: Simplify

$$\frac{x^2 - 3x - 10}{x^2 - 2x - 15} = \frac{\underset{\text{Factor}}{(x-5)(x+2)}}{\underset{\text{Divide}}{(x-5)(x+3)}} = \frac{x+2}{x+3}$$