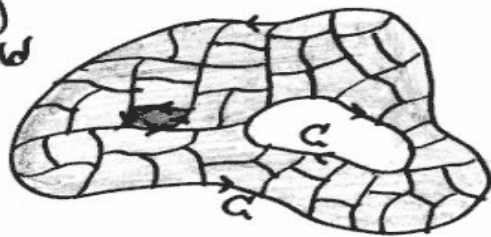


$\vec{F}$  (uniformly continuously) differentiable vector field

$R$  region bounded by piecewise smooth curve  $C$



$$\oint_C \vec{F} \cdot d\vec{r} \stackrel{!}{=} \sum_K \oint_{C_K} \vec{F} \cdot d\vec{r}$$

$\vec{F}$  is differentiable!  $\rightarrow \approx$

$$\sum_K \oint_{C_K} \vec{L}_K \cdot d\vec{r}$$

Green's thm  
for linear  
fields.  
(PRECALC!)  $\stackrel{!}{=}$

$$\sum_K (c_K - b_K) \cdot (\text{area of } R_K)$$

$\rightarrow \approx$

$$\sum_K \left( \frac{\partial F_2}{\partial x}(p_K) - \frac{\partial F_1}{\partial y}(p_K) \right) \cdot (\text{area of } R_K)$$

defn of integral  
R-sums.  $\rightarrow \approx$

$$\iint_R \left( \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} \right) dA$$

Straightforward exercise to estimate the errors made in the approximations, and to show that in the limit, as  $\text{length}(C_K) \rightarrow 0$  approximate equalities become indeed equal.