## 1-3 Continuity, End Behavior, and Limits — A Exercises 1-23

Determine whether each function is continuous at the given x-value(s). Justify using the continuity test. If discontinuous, identify the type of discontinuity as removable, jump, or infinite. Justify.

1. $f(x) = \dfrac{x^2 - 9}{x - 3}$

$f(3) = \dfrac{3^2 - 9}{3 - 3} = \dfrac{0}{0}$

$= \dfrac{0}{0}$

$= \text{UND}$

The function is defined at $x = 3$.

Find $\lim_{x \to 3} f(x)$. Construct a table that shows values of $f(x)$ for $x$-values approaching $x = 3$ from the left and from the right.

| $x$ | 2.9 | 2.99 | 2.999 | 3 | 3.001 | 3.01 | 3.1 |
|---|---|---|---|---|---|---|---|
| $f(x)$ | 5.9 | 5.99 | 5.999 | | 6.001 | 6.01 | 6.1 |

$\lim_{x \to 3} f(x) = 6$

Because $\lim_{x \to 3} f(x) \neq f(3)$, $f(x)$ is discontinuous at $x = 3$.

2. $f(x) = \dfrac{\sqrt{x} - 2}{x - 4}$

$f(4) = \dfrac{\sqrt{4} - 2}{4 - 4}$

$= \dfrac{0}{0}$

$= \text{UND}$

The function is defined at $x = 4$.

Find $\lim_{x \to 4} f(x)$. Construct a table that shows values of $f(x)$ for $x$-values approaching 4 from the left and from the right.

| $x$ | 3.9 | 3.99 | 3.999 | 4 | 4.001 | 4.01 | 4.1 |
|---|---|---|---|---|---|---|---|
| $f(x)$ | 0.24 | 0.25 | 0.252 | | 0.248 | 0.24 | 0.227 |

$\lim_{x \to 4} f(x) = 0.25$

Because $\lim_{x \to 4} f(x) = f(4)$, $f(x)$ is continuous at $x = 4$.