Conditional Probability & Independence

The general formula for determining the probability of an event is:

$$P(\text{event}) = \frac{\text{the total number of successes}}{\text{the total number of outcomes}}$$

This is still true even if I tell you some information about the outcome before you calculate the probability. These sorts of problems involve **conditional probability**. The given information in a problem reduces the sample space and changes the value of the denominator of the fraction in the general formula above, and sometimes the numerator's value as well. We might describe a conditional probability as "the chances that A happens, given that B happens", and the notation for that is P(A|B).

Example 1: I draw a red card from a well-shuffled deck of cards. What is the probability that the card I drew was (a) a heart, (b) a king, and (c) a club?

Solution: There are 52 cards in a deck of cards, but only 26 of them are red. Since we know that the card that was drawn is a red card, the total number of outcomes gets reduced from 52 to 26.

(a) There are 13 hearts in the deck, all of them red, so the number of successes doesn't change. P(heart | red card) = $\frac{13}{26} = \frac{1}{2}$.

(b) There are 4 kings in the deck, but two of them are not in the sample space anymore because they are black cards, so there are only two successes in this case. P(king | red card) = $\frac{2}{26} = \frac{1}{13}$. (Notice that this is the same probability as drawing a king without being told the colour of the card.)

(c) Since clubs are black cards, not only is the denominator of the fraction changed, the numerator is too. All of the successes have been removed from the sample space, so P(club | red card) = $\frac{0}{26} = 0$.

In situations where the outcomes are not countable, we can calculate a conditional probability from other probabilities.

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$$

This formula works on countable probabilities as well; for the card example above, we would have had $\frac{13/52}{26/52}$, $\frac{2/52}{26/52}$ and $\frac{0/52}{26/52}$, which would reduce to the probabilities we calculated anyway since all those "over 52"s would have cancelled out.

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