

18.5 Strategy

Students sometimes submit the matrix A and b and solve for x by minimizing $\|Ax - b\|_2$.

These different formulations for least-squares with $\|A\|_2 = 1$

are related, and have connections to results in the $\|A\|_2 = 1$ setting for the following three special cases below.

(1)

(2)

(3)

Case 1: Rank 1 Matrices with $\|A\|_2 = 1$

Consider A to be a rank 1 matrix with $\|A\|_2 = 1$. The SVD decomposition of A is $A = U \Sigma V^T$, where U and V are orthogonal matrices and Σ is a diagonal matrix.

Derivation of the Solution for Case 1: Rank 1

Using the SVD decomposition of A , we can write $Ax - b = U \Sigma V^T x - b$.

Since U is orthogonal, $\|U \Sigma V^T x - b\|_2 = \|\Sigma V^T x - U^T b\|_2$.

Let $y = V^T x$ and $c = U^T b$. Then the problem becomes $\|\Sigma y - c\|_2$.

Since Σ is a diagonal matrix, the problem is decoupled into n independent 1D problems.

Case 2: Rank 2 Matrices with $\|A\|_2 = 1$

If A is a rank 2 matrix with $\|A\|_2 = 1$, the SVD decomposition is $A = U \Sigma V^T$, where U and V are orthogonal matrices and Σ is a diagonal matrix with two non-zero entries.

Derivation of the Solution for Case 2: Rank 2

Using the SVD decomposition of A , we can write $Ax - b = U \Sigma V^T x - b$.

Since U is orthogonal, $\|U \Sigma V^T x - b\|_2 = \|\Sigma V^T x - U^T b\|_2$.

Let $y = V^T x$ and $c = U^T b$. Then the problem becomes $\|\Sigma y - c\|_2$.

Since Σ is a diagonal matrix, the problem is decoupled into n independent 1D problems.

18.6 AC and the QR decomposition