Solve, giving your answers correct to 3 decimal places, (a) $3^{x+1} = 15$, (b) $(0{\cdot}8)^{2x} = 0{\cdot}45$.

(a)

$$3^{x+1} = 15$$
$$\therefore \quad \log 3^{x+1} = \log 15$$
$$\therefore \quad (x+1)\log 3 = \log 15$$
$$\therefore \quad x+1 = \frac{\log 15}{\log 3}$$
$$x = \frac{\log 15}{\log 3} - 1$$
$$\text{or} \quad x = 1{\cdot}465$$

(b)

$$(0{\cdot}8)^{2x} = 0{\cdot}45$$
$$\therefore \quad \log(0{\cdot}8)^{2x} = \log 0{\cdot}45$$
$$\therefore \quad 2x\log(0{\cdot}8) = \log 0{\cdot}45$$
$$\therefore \quad 2x = \frac{\log 0{\cdot}45}{\log 0{\cdot}8}$$
$$= 3{\cdot}5784$$
$$\text{or} \quad x = 1{\cdot}789$$

Example 10

Solve for x the equations (a) $5^{2x+1} + 4 = 21 \times 5^x$, (b) $\log_3 x + \log_x 9 = 3$.

(a)

$$5^{2x+1} + 4 = 21 \times 5^x$$
$$\text{or} \quad 5 \times 5^{2x} - 21 \times 5^x + 4 = 0$$

letting $y = 5^x$,

$$5y^2 - 21y + 4 = 0$$
$$(5y - 1)(y - 4) = 0$$
$$y = \tfrac{1}{5} \quad \text{or} \quad y = 4$$
$$\therefore \quad 5^x = \tfrac{1}{5} \quad \text{or} \quad 5^x = 4$$
$$= 5^{-1} \qquad x\log 5 = \log 4$$
$$\text{i.e.} \quad x = -1 \quad \text{or} \quad x = \frac{\log 4}{\log 5} = 0{\cdot}86$$

(b)

$$\log_3 x + \log_x 9 = 3$$
$$\log_3 x + \frac{\log_3 9}{\log_3 x} = 3$$
$$(\log_3 x)^2 + 2 = 3\log_3 x$$

letting $y = \log_3 x$,

$$y^2 - 3y + 2 = 0$$
$$(y - 1)(y - 2) = 0$$
$$y = 1 \quad \text{or} \quad y = 2$$
$$\therefore \quad \log_3 x = 1 \quad \text{or} \quad \log_3 x = 2$$
$$\text{i.e.} \quad x = 3 \quad \text{or} \quad x = 9$$