A Shape Problem

How many points with **integer coordinates** are there on the **boundary** of the n-dimensional generalisation of a triangle (ie. triangle, pyramid, ...) with the following conditions;

Case 1: Vertices $(0, 0, ..., 0)$ and $x_i = (0, ..., 0, \frac{2n}{i}, 0, ..., 0)$ for $1 \leq i \leq n$.

Case 2: Vertices $(0, 0, ..., 0)$ and $x_i = (0, ..., 0, \frac{2n+1}{i}, 0, ..., 0)$ for $1 \leq i \leq n$.

where x_i has the non-zero coordinate in the i^{th} spot. Think of them as even and odd cases. I realised halfway through writing this that it's a little confusing, so here's what I mean in more detail;

Example 1: Triangle

$n = 2$. Case 1 has vertices $(0, 0), (4, 0)$ and $(0, 2)$. Case 2 has vertices $(0, 0), (5, 0)$ and $(0, \frac{5}{2})$ We're looking for all points with integer coordinates lying on vertices or edges.

Example 2: Pyramid

$n = 3$. Case 1 has vertices $(0, 0, 0), (6, 0, 0), (0, 3, 0)$ and $(0, 0, 2)$. Case 2 has vertices $(0, 0, 0), (7, 0, 0), (0, \frac{7}{2}, 0)$ and $(0, 0, \frac{7}{3})$. We're looking for all points with integer coordinates lying on vertices or edges **or faces**.